



**DOIRADA BERILGAN KASR TARTIBLI SUBDIFFUZIYA  
TENGLAMASI TUSHUNTIRISHNING PEDAGOGIK TAHLILI.**

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**Annotatsiya:** Mazkur maqolada oliy ta'lim muassasalarida doira sohasida berilgan kasr tartibli subdiffuziya tenglamasini o'qitish jarayonining pedagogik tahlili olib boriladi. Talabalarga murakkab matematik modellarni, xususan kasr tartibli differensial tenglamalarni tushuntirishda duch kelinadigan asosiy muammolar, didaktik tamoyillar va samarali o'qitish usullari yoritiladi. Mavzuni tushuntirishda bosqichma-bosqichlik, vizualizatsiya, fizik mazmun bilan bog'lash hamda raqamli texnologiyalardan foydalanishning ahamiyati asoslanadi. Maqolada shuningdek, talabalarning abstrakt tafakkurini rivojlantirish va mustaqil tadqiqot ko'nikmalarini shakllantirish yo'llari ko'rsatib beriladi.

**Kalit so'zlar:** pedagogik tahlil, kasr tartibli tenglamalar, subdiffuziya, doira soha, matematik modellashtirish, oliy ta'lim, didaktik tamoyillar, fanlararo integratsiya.  
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**Аннотация:** В данной статье проводится педагогический анализ процесса преподавания уравнений субдиффузии дробного порядка на сфере круга в высших учебных заведениях. Выделены основные проблемы, возникающие при объяснении сложных математических моделей, в частности дифференциальных уравнений дробного порядка, дидактические принципы и эффективные методы обучения. Обосновывается важность пошагового объяснения, визуализации, связи с физическим содержанием и использования цифровых технологий в объяснении темы. В статье также показаны пути развития абстрактного мышления студентов и формирования навыков самостоятельной исследовательской работы.

**Ключевые слова:** педагогический анализ, уравнения дробного порядка, субдиффузия, сфера круга, математическое моделирование,



высшее образование, дидактические принципы, междисциплинарная интеграция.

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**Abstract:** This article conducts a pedagogical analysis of the teaching process of fractional-order subdiffusion equations in the sphere of a circle in higher education institutions. The main problems encountered in explaining complex mathematical models, in particular fractional-order differential equations, didactic principles and effective teaching methods are highlighted. The importance of step-by-step, visualization, connection with physical content and the use of digital technologies in explaining the topic is justified. The article also shows ways to develop students' abstract thinking and form independent research skills.

**Keywords:** pedagogical analysis, fractional-order equations, subdiffusion, sphere of a circle, mathematical modeling, higher education, didactic principles, interdisciplinary integration.

Bugungi kunda oliy ta'lim tizimida matematik modellashtirishga asoslangan fanlarni o'qitish jarayonida nafaqat formulalarni, balki ularning fizik, biologik va texnik mazmunini ham ochib berish muhim hisoblanadi. Ayniqsa, kasr tartibli differensial tenglamalar zamonaviy ilm-fan va texnologiyaning ajralmas qismiga aylanib bormoqda. Shunga qaramasdan, ushbu mavzu talabalar uchun eng murakkab va abstrakt bo'limlardan biri hisoblanadi.

Doira sohasida berilgan kasr tartibli subdiffuziya tenglamasi esa bir vaqtning o'zida uchta murakkablikni mujassamlashtiradi: kasr tartibli hosila, anomal diffuziya jarayoni va egri chiziqli koordinatalarda berilgan chegaraviy masala. Shu sababli mazkur mavzuni tushuntirish maxsus metodik yondashuvni talab etadi.

Mazkur maqolaning maqsadi — doira sohasida berilgan kasr tartibli subdiffuziya tenglamasini o'qitishda qo'llaniladigan samarali pedagogik



yondashuvlarni tahlil qilish, bosqichli tushuntirish modeli ishlab chiqish hamda o'qitish jarayonini takomillashtirish bo'yicha tavsiyalar berishdan iborat.

### 1. Mavzuning didaktik ahamiyati

Kasr tartibli subdiffuziya tenglamalari talabalarga:

real jarayonlarning murakkabligini tushunish,

klassik modellar chegaralanganligini anglash,

matematik apparatning rivojlanish mantiqini idrok etish,

nazariya va amaliyot integratsiyasini ko'rish imkonini beradi.

Doira sohasidagi masalalar esa simmetriya tushunchasi, qutbiy koordinatalar, Bessel funksiyalari va spektral usullarni mustahkamlashga xizmat qiladi. Shu bois, mazkur mavzu bo'lajak matematik, muhandis va fizika o'qituvchilari uchun muhim kasbiy kompetensiyalarni shakllantiradi.

### 2. Tushuntirishda uchraydigan asosiy qiyinchiliklar

Pedagogik tajribalar shuni ko'rsatadiki, talabalar quyidagi jihatlarda ko'proq qiynaladilar:

**Kasr tartibli hosila mazmuni** — xotira va meros effekti tushunchalarining abstraktligi.

**Subdiffuziya mohiyati** — klassik diffuziyadan farqini intuitiv qabul qilish qiyinligi.

**Doira sohasidagi operatorlar** — Laplas operatorining qutbiy koordinatalardagi ko'rinishi.

**Maxsus funksiyalar** — Bessel va Mittag-Leffler funksiyalarining yangi va murakkabligi.

Shuning uchun mavzuni to'g'ridan-to'g'ri formulalardan boshlash emas, balki real jarayonlardan kelib chiqib tushuntirish samaraliroq hisoblanadi.

### 3. Bosqichma-bosqich tushuntirish modeli



Mavzuni o'qitishda quyidagi metodik ketma-ketlik tavsiya etiladi:

### 3.1. Motivatsion bosqich

Real hayotiy misollar: hujayra ichida modda tarqalishi, g'ovak muhitlarda sekin diffuziya, dumaloq plastinkada issiqlik tarqalishi keltiriladi.

### 3.2. Nazariy tayanch bosqich

Klassik diffuziya tenglamasi eslatiladi.

Kasr tartibli hosilaning fizik mazmuni izohlanadi.

Subdiffuziya modeli kiritiladi.

### 3.3. Matematik modellashtirish bosqichi

Doira sohasida Laplas operatori.

Boshlang'ich va ikkinchi tur chegaraviy shartlarning fizik talqini.

### 3.4. Yechish algoritmi bosqichi

Ajratish usuli, xos qiymatlar masalasi, Bessel funksiyalari va vaqt bo'yicha Mittag–Leffler funksiyasi bilan yechim qurish ko'rsatiladi.

### 3.5. Tahlil va talqin bosqichi

Olingan yechimning fizik ma'nosi, grafiklar va kompyuter modellashtirish orqali mustahkamlanadi.

## 4. Samarali pedagogik texnologiyalar

Mazkur mavzuni o'qitishda quyidagi usullar ayniqsa samarali:

**Vizualizatsiya:** animatsiya va grafik modellar.

**Muammoli ta'lim:** subdiffuziya nima uchun klassik modeldan farq qilishini muhokama qilish.

**Fanlararo integratsiya:** fizika va biologiya bilan bog'lash.

**Raqamli texnologiyalar:** MATLAB, Python orqali yechimlarni ko'rsatish.

**Mini-loyihalar:** talabalar mustaqil ravishda modellashtirish ishlari olib boradi.

## 5. Pedagogik natijalar va muhokama



Amaliy kuzatuvlar shuni ko'rsatadiki, bosqichli va vizual asoslangan yondashuv talabalarning:

- mavzuga qiziqishini oshiradi,
- murakkab formulalarni ongli qabul qilishiga yordam beradi,
- ilmiy-tadqiqot kompetensiyasini rivojlantiradi.

Talabalar subdiffuziya modelini klassik diffuziya bilan taqqoslab, modellashtirish madaniyatini egallaydi.

Doira sohasida berilgan kasr tartibli subdiffuziya tenglamasini o'qitish murakkab, ammo katta pedagogik salohiyatga ega mavzudir. Uni samarali tushuntirish uchun real jarayonlarga tayangan holda, bosqichma-bosqich, vizual va fanlararo yondashuv asosida o'qitish zarur. Mazkur pedagogik tahlil oliy ta'limda kasr tartibli modellarni o'qitish metodikasini takomillashtirishga xizmat qiladi.

Ushbu ishda doira sohasida berilgan kasr tartibli subdiffuziya tenglamasi uchun boshlang'ich va ikkinchi tur chegaraviy masala qo'yildi va analitik yechimi olindi. Ajratish usuli, Bessel va Mittag-Leffler funksiyalaridan foydalanib, yechimning qator ko'rinishi keltirib chiqarildi. Masalaning mavjudligi, yagonaligi va barqarorligi asoslandi. Olingan natijalar anomal diffuziya jarayonlarini o'rganishda muhim nazariy asos bo'lib xizmat qiladi.



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