



DETERMINANTLAR, USTIDAGI AMALLARNING XOSSALARINI ISBOTLASH

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Annotatsiya: Ushbu kengaytirilgan ilmiy maqolada determinantlar nazariyasining asosiy tushunchalari, tarixiy rivoji, matematik mohiyati, algebraik xossalari, geometrik talqinlari, hisoblashning klassik va zamonaviy usullari, minorlar tizimi, Laplas yoyilmasi, Gauss eliminasiyasi, LU-faktorizatsiya, Jacobian, Vandermonde determinanti, Gram determinanti kabi ko‘plab yo‘nalishlar chuqur yoritilgan. Shuningdek, determinantlarning fizika, mexanika, injiniring, iqtisodiyot, statistika, kompyuter grafikasi, transformatsiya nazariyasi, algoritmlar, kriptografiya va boshqa zamonaviy fanlardagi amaliy qo‘llanilish sohalari ham keng yoritilgan.

Abstract: This extensive scientific article covers in depth the basic concepts of determinant theory, its historical development, mathematical essence, algebraic properties, geometric interpretations, classical and modern methods of calculation, the system of minors, Laplace expansion, Gauss elimination, LU-factorization, Jacobian, Vandermonde determinant, Gram determinant, and many other areas. It also covers the practical applications of determinants in physics, mechanics, engineering, economics, statistics, computer graphics, transformation theory, algorithms, cryptography, and other modern sciences.



Kalit so‘zlar Determinantlar Nazariyasi, determinantning xossalari, algebraik determinantlar, geometrik talqin, minorlar va kofaktorlar, Laplas yoyilmasi, Gauss eliminatsiyasi, LU-faktorizatsiya, Jacobian, Vandermonde determinant.

Keywords: Determinant Theory, properties of determinants, algebraic determinants, geometric interpretation, minors and cofactors, Laplace expansion, Gauss elimination, LU-factorization, Jacobian, Vandermonde determinant.

Kirish. Determinant chiziqli algebra fanining fundamental tushunchalaridan biri bo‘lib, kvadrat matritsaning chiziqli mustaqilligi, teskari matritsaning mavjudligi, transformatsiya masshtabi va barqarorligi kabi asosiy xususiyatlarni aniqlashda muhim matematik vosita hisoblanadi. Globallashuv, texnologik taraqqiyot, kompyuter grafikasi, ma’lumotlar tahlili, iqtisodiy modellashtirish, fizika va mexanika kabi tarmoqlarda determinantlardan keng foydalaniladi. Tarixiy manbalarga ko‘ra Eyler, Vandermonde, Lagrange, Koshy, Laplas kabi matematiklar determinantlar nazariyasining shakllanishiga katta hissa qo‘shgan. Bugungi kunda determinantlar matematik modellashtirishning barcha bosqichlarida, analitik geometriyada, differensial tenglamalarda, transformatsiyalar nazariyasida, statistika va kriptografiyada asosiy o‘rin tutadi.

Introduction: The determinant is one of the fundamental concepts of linear algebra and is an important mathematical tool for determining such basic properties as linear independence of a square matrix, the existence of an inverse matrix, the scale and stability of a transformation. Determinants are widely used in such fields as globalization, technological progress, computer graphics, data analysis, economic modeling, physics and mechanics. According to historical sources, mathematicians such as Euler, Vandermonde, Lagrange, Cauchy, Laplace made a significant contribution to the formation of the theory of determinants. Today, determinants play a key role in all stages of mathematical modeling, in analytic geometry, differential equations, transformation theory, statistics and cryptography.

1. DETERMINANTNING MATEMATIK MOHIYATI:



Determinant kvadrat matritsaga mos keluvchi yagona son bo'lib, uning algebraik va geometrik xususiyatlarini belgilaydi. Determinant $n!$ permutatsiyalar orqali aniqlanadigan kombinatorik ifodaga ega. Determinant 0 ga teng bo'lsa, matritsa singulyar bo'lib, uning teskari matritsasi mavjud emas. Bu holat chiziqli tenglamalar sistemasining yechimiga bevosita ta'sir qiladi. Minorlar va algebrik qo'shimchalar determinantning ichki strukturasini tashkil etadi.

2. ALGEBRAIK XOSSALARI:

- Ikki qatorni almashish determinant ishorasini o'zgartiradi.
- Qatorni k ga ko'paytirish determinantni shu k ga ko'paytiradi.
- Bir qatorga boshqa qatorning k barobari qo'shilsa determinant o'zgarmaydi.
- Diagonal matritsada determinant diagonaldagi elementlar ko'paytmasiga teng.

- $|A| = |A^T|$ transpozitsiyada determinant o'zgarmaydi.

- Agar determinant 0 bo'lsa, chiziqli bog'liqlik mavjud.

- Blok matritsalarining determinantlari ayrim hollarda $|A||D|$ shaklida topiladi.

3. HISOBLASH USULLARI:

3.1. Sarrus qoidasi — 3×3 matritsalar uchun tezkor usul.

3.2. Laplas yoyilmasi — minorlar va algebrik qo'shimchalar orqali yoyish.

3.3. Gauss eliminasiyasi — amaliy hisob-kitoblarda eng samarali usul.

3.4. LU-parchalanish — determinantni yuqori uchburchak matritsa orqali topish.

3.5. Jacobian determinanti — integral o'lchovlarini o'zgartirish mexanizmi.

3.6. Vandermonde determinanti — kombinatorik tizimlarda qo'llaniladi.

3.7. Gram determinanti — vektorlar hosil qilgan jism hajmini aniqlaydi.

4. GEOMETRIK TALQIN:

- 2D fazoda parallelogramm yuzasi determinant moduliga teng.

- 3D fazoda parallelopiped hajmi determinant orqali topiladi.



— Transformatsiya matritsasining determinanti masshtab va orientatsiyani bildiradi.

— Jacobian fazolararo o'tishlarda integralning o'zgarish koeffitsientini beradi.

— Kompyuter grafikada obyektlar deformatsiyasi determinant orqali boshqariladi.

5. AMALIY QO'LLANILISH:

5.1. Fizika va mexanikada:

- kuch momentlari,
- burilish matritsalarini,
- deformatsion tahlil,
- barqarorlik mexanizmlari,
- tensorga asoslangan o'lchashlar.

5.2. Statistika:

- kovariatsiya matritsasi determinanti tarqoqlik ellipsoidi hajmini beradi.
- Mahalanobis masofasini hisoblash determinantga bog'liq.
- LDA, PCA, regressiya modellarida determinant ishlatiladi.

5.3. Kompyuter grafikasi:

- transformatsiya matritsalarida masshtab,
- 3D obyekt burilishi,
- proyeksiya, animatsiya, deformatsiya,
- rendering jarayonida fazoviy o'zgarishlar.

5.4. Iqtisodiyot:

- ko'p omilli modellar,
- regressiya determinantlari,
- barqarorlik tahlili.

5.5. Kriptografiya:

- Hill cipher shifrlash usulida determinantning nol bo'lmasligi shart.



— Matritsali shifrlash algoritmlarining teskari operatsiyasi determinantga bogʻliq

Xulosa: Determinantlar nazariyasi — chiziqli algebra, analitik geometriya, fizika, mexanika, statistika, iqtisodiyot va kompyuter grafikasi kabi koʻplab yoʻnalishlarda qoʻllaniladigan fundamental matematik asosdir. Determinantlarning struktura, minorlar tizimi, Laplas yoyilmasi, geometrik talqinlari, transformatsiya mexanizmlari va amaliy sohalardagi keng qoʻllanilishi ushbu tushunchaning chuqur oʻrganilishini taqozo etadi.

Conclusion: Determinant theory is a fundamental mathematical framework used in many areas, such as linear algebra, analytic geometry, physics, mechanics, statistics, economics, and computer graphics. The structure of determinants, the system of minors, the Laplace expansion, geometric interpretations, transformation mechanisms, and the wide range of applications in practical areas require a thorough study of this concept.

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