PROBLEMS OF MATHEMATICAL MODELING OF THE INTERDEPENDENCE OF ECONOMIC SECTORS

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Abstract: This scientific article studies the problems of mathematical modeling of the interdependence of economic sectors across the country. The relevance of the article is that if the state plans to increase the volume of final products in one sector (for example, industry), it is impossible to calculate how much additional costs this will require in other adjacent sectors (energy, transport, agriculture) without accurate mathematical matrices. To solve the problem, V. Leontiev's "Cost - Production" inter-sectoral balance model was used. In the practical part, direct costs and Leontiev's full cost matrix were calculated on the example of a two-sector conditional economy. The results of the study show that the state's target programs for economic growth can achieve a real balance only if they are based on gross production volumes obtained using inverse matrices.

Keywords: inter-sectoral balance, Leontiev model, cost-production, macroeconomic equilibrium, direct cost coefficient, full cost matrix, mathematical modeling.

INTRODUCTION

The modern economy consists of a complex chain of hundreds of sectors and sub-sectors. For example, to increase the production of cars in the country, more metal is needed; more electricity is consumed to melt metal; and to produce electricity, more gas is needed to extract and transport services to deliver it. If the state budget or strategic programs focus only on the automotive industry, and the mathematical needs of other related sectors are not taken into account, the economy will experience a deficit and a chain crisis.

The problem raised in the article is precisely to balance such "hidden and multiple" needs in the economy using mathematical calculations. The relevance of the research is that at a time when large-scale industry and clustered agriculture are developing in Uzbekistan, investment strategies must be built on the basis of a strict mathematical balance. The purpose of the work is to construct a system of linear algebraic equations of the inter-industry balance (Input-Output) model, apply the concept of an inverse matrix to the economy, and prove numerically how the economy adapts when the state order changes.

LITERATURE REVIEW

The idea of expressing the interrelationships of sectors in the economy in the form of mathematical matrices was developed in the 1930s by the Russian-American economist Vasily Leontief. He was awarded the Nobel Prize in 1973 for compiling large tables of the US economy and making a fundamental change in economic planning. This "Input-Output" model is still considered the most powerful tool for macroeconomic planning worldwide.

RESEARCH METHODOLOGY

Leontiev's inter-sectoral balance model studies the economy by dividing it into n sectors. The main variables in the model are:

X_i —total gross output of i- sector;

Y_i — volume of output intended for final consumption of i--sector (consumption of the population, exports, government purchases);

a_{ij} -is the direct cost coefficient. It shows how much output the j-industry needs to purchase from the □i-industry to produce 1 unit of output.

The basic balance equation of the model is written as follows for each industry:

$$X_i = \sum_{j=1}^{n} a_{ij}X_j + Y_i$$

This means that the total output produced by each sector (X_i) is equal to the portion it sells as raw materials to other sectors ($\sum a_{ij}X_j$) and the portion it sells directly to final consumers (Y_i).

We write this system of equations in matrix form:

$$X = AX + Y$$

Here A is the direct cost matrix, X is the vector of gross products, and Y is the vector of final products.

From this equation, we perform mathematical substitutions to find the gross output (X):

$$X - AX = Y \Longrightarrow (E - A)X = Y$$

$$X = (E - A)^{-1}Y$$

where E is the identity matrix, and $(E - A)^{-1}$ is called the Leontief full cost (inverse) matrix. If the state confirms its final target vector Y, then by multiplying it by this inverse matrix, it is precisely calculated what aggregate production plan (X) should be given to the sectors in the entire economy.

DISCUSSION AND RESULTS

To demonstrate the logic of the algebraic formulas in the methodology, we conditionally divide the country's economy into 2 large sectors: Sector 1 (Industry) and Sector 2 (Agriculture).

As a result of the research, it was found that the direct cost matrix (A) in the economy is as follows:

$$A = \begin{pmatrix} 0.2 & 0.4 \\ 0.3 & 0.1 \end{pmatrix}$$

This means that in order for Agriculture to produce 1 trillion soums worth of products, it must purchase 0.4 trillion soums worth of equipment/fertilizers from Industry and 0.1 trillion soums worth of seeds from its own network.

The government has drawn up a final consumption plan (Y) for the coming year:

Final demand for industrial products: $Y_1 = 200$ trillion soums.

Final demand for agricultural products: $Y_2 = 100$ trillion soums.

Task: Calculate how much total gross product $(X_1$ and $X_2)$ must be produced by Industry and Agriculture to satisfy this final consumption.

Step 1. We find the matrix $(E - A)$:

$$(E - A) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.2 & 0.4 \\ 0.3 & 0.1 \end{pmatrix} = \begin{pmatrix} 0.8 & -0.4 \\ -0.3 & 0.9 \end{pmatrix}$$

Step 2. Calculate the determinant of the matrix $(E - A)$:

$$\Delta = (0.8 \times 0.9) - (-0.4 \times -0.3) = 0.72 - 0.12 = 0.60$$

Step 3. We find the Leontief full cost inverse matrix $(E - A)^{-1}$:

$$(E - A)^{-1} = \frac{1}{0.6}\begin{pmatrix} 0.9 & 0.4 \\ 0.3 & 0.8 \end{pmatrix} = \begin{pmatrix} 1.5 & 0.67 \\ 0.5 & 1.33 \end{pmatrix}$$

Step 4. We calculate the gross production vector ($X = (E - A)^{-1}Y$:

$$X = \begin{pmatrix} 1.5 & 0.67 \\ 0.5 & 1.33 \end{pmatrix}\begin{pmatrix} 200 \\ 100 \end{pmatrix} = \begin{pmatrix} 1.5 \times 200 + 0.67 \times 100 \\ 0.5 \times 200 + 1.33 \times 100 \end{pmatrix} = \begin{pmatrix} 300 + 67 \\ 100 + 133 \end{pmatrix} = \begin{pmatrix} 367 \\ 233 \end{pmatrix}$$

Let's systematize the obtained mathematical results:

Table 1

THE ECONOMY'S GROSS PRODUCTION PLAN TO SATISFY FINAL DEMAND [1]

Networks	**Final government demand (Y), trillion**	**Chain spending within the network, trillion**	**Total Gross Domestic Product (X) Demand, trillion**
Industry	200	167	**367**
Agriculture	100	133	**233**

Networks	**Final government demand (Y), trillion**	**Chain spending within the network, trillion**	**Total Gross Domestic Product (X) Demand, trillion**
Total economy:	**300 trillion soums**	**300 trillion soums**	**600 trillion soums**

CONCLUSIONS AND SUGGESTIONS

Based on the conducted inter-sectoral balance modeling and matrix analysis, the following important conclusions were drawn:

1. If state economists were unaware of Leontiev's mathematical model, they would have made a gross mistake by setting a total plan of 300 trillion soums for the supply of 200 trillion industrial and 100 trillion agricultural products. Because as can be seen from the table, another 300 trillion soums of production is needed to cover the chain (hidden) needs within the economy in order to reach the final goal of 300 trillion. The actual target plan that maintains the balance is 600 trillion soums.

2. The elements of the inverse matrix play the role of a multiplier. The coefficient of 1.5 in our mathematical calculation means that in order to increase the final product of the Industry itself by 1 trillion soums, it will be necessary to increase the gross production of this sector by at least 1.5 trillion soums.

3. It is strongly recommended that macroeconomic and regional research institutes constantly update Leontief matrices between more than 30 sectors of the national economy of Uzbekistan and simulate in advance on the basis of this model

what "shocks" (shortages) subsidies introduced in any sector will cause in other sectors.

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