MAVZU: TO‘LA DIFFERENSIAL TENGLAMA, KLERO DIFFERENSIAL TENGLAMASI VA LAGRANJ DIFFERENSIAL TENGLAMASI

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Ta'rif. Agar

$$\boldsymbol{M(x,y)dx + N(x,y)dy = 0} \ (*)$$

Tenglamada $M(x,y)$, va $N(x,y)$ funksiyalar uzluksiz,differensiallanuvchi

bo‘lib,bular uchun

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

munosabat bajarilsa, (*) tenglama ***to‘la differensial tenglama*** deyiladi,

bunda

$$\frac{\partial M}{\partial y} \text{ va } \frac{\partial N}{\partial x}$$

funksiyalar biror sohada uzluksiz funksiyalardir.

Misol . Ushbu tenglama berilgan :

$$\frac{2x}{y^3}dx + \frac{y^2 - 3x^2}{y^4}dy = 0.$$

Bu tenglamaning to‘la differensial tenglama bo‘lish yoki bo‘lmasligini tekshiramiz.

Bu yerda

$$M = \frac{2x}{y^3}, \quad N = \frac{y^2 - 3x^2}{y^4}$$

deb olamiz ,bu holda

$$\frac{\partial M}{\partial y} = -\frac{6x}{y^4}, \quad \frac{\partial N}{\partial x} = -\frac{6x}{y^4}$$

$y \neq 0$ bo‘lganida shart bajariladi . Demak berilgan tenglamaning chap tomoni biror noma’lum *u(x,y)* funksiyaning to‘la differensiali bo‘ladi.

Bu funksiyani topamiz .

$$\frac{\partial u}{\partial x} = \frac{2x}{y^3}$$

bo‘lganligi sababli

$$u = \int \frac{2x}{y^3} dx + \varphi(y) = \frac{x^2}{y^3} + \varphi(y),$$

$$\frac{\partial u}{\partial y} = N = \frac{y^2 - 3x^2}{y^4}$$

ekanligini e’tiborga olib ,

$$-\frac{3x^2}{y^4} + \varphi'^{(y)} = \frac{y^2 - 3x^2}{y^4},$$

bo‘lishini topamiz. Demak,

$$\varphi'^{(y)} = \frac{1}{y^2} \ , \qquad \varphi(y) = -\frac{1}{y} + C_1$$

$$u(x,y) = \frac{x^2}{y^3} - \frac{1}{y} + C_1$$

Shunday qilib,dastlabki tenglamaning umumiy integralini topamiz.

Lagranj differensial tenglamasi

Ta'rif. ***x*** va ***y*** ga nisbatan chiziqli bo'lgan koeffitsiyentlari esa y' ning funksiyalari bo'lgan ushbu

$$y = x \cdot \varphi(y') + \Phi(y')$$

differensial tenglamaga **Lagranj differensial tenglamasi** deyiladi.

Ushbu tenglamani yechish algoritmi quyidagicha:

1) Umumiy yechimni topish uchun $p = y'$ o'zgaruvchi almashtiriladi.

Differensial tenglama quyidagicha ko'rinishga keltiriladi:

$$y = x \cdot \varphi(p) + \Phi(p)$$

2) Ushbu tenglamani $y' = p \;\Rightarrow\; dy = pdx$ ekanligini e'tiborga olib differensiallaymiz.

$$dy = d\big(x \cdot \varphi(p) + \Phi(p)\big) \;\Rightarrow$$
$$pdx = \varphi(p)dx + x \cdot \varphi'(p)dp + \Phi'(p)dp$$
$$\frac{dx}{dp} - \frac{\varphi'(p)}{p - \varphi(p)}x = \frac{\Phi'(p)}{p - \varphi(p)}$$

chiziqli differensial tenglamani hosil qilamiz.

Bu yerda $p - \varphi(p) = 0$ yechim alohida aniqlanadi.

3) *x* ga nisbatan chiziqli bo'lgan bu differensial tenglamaning yechimi *x=F(p,c)* bo'lsa, u holda Lagranj differensial tenglamasining umumiy yechimi quyidagicha bo'ladi:

$$\begin{cases} x = F(p,c) \\ y = x \cdot \varphi(p) + \Phi(p) = F(p,c) \cdot \varphi(p) + \Phi(p) \end{cases}.$$

Misol . $y = 2x(y)' - 4y'^3$

differensial tenglamaning umumiy yechimini toping.

Yechish. $y' = p \quad \Rightarrow \quad y = 2xp - 4p^3$ tenglikni differensiallasak,

$$y' = 2p + 2x\frac{dp}{dx} - 12p^2\frac{dp}{dx}$$

$$p = 2p + 2x\frac{dp}{dx} - 12p^2\frac{dp}{dx}$$

$$12p^2\frac{dp}{dx} = p + 2x\frac{dp}{dx}$$

tenglikni ikkala tomonini $\dfrac{dp}{dx}$ bo‘lamiz:

$$12p^2 = p\frac{dx}{dp} + 2x$$

$$p\frac{dx}{dp} + 2x = 12p^2$$

$$\frac{dx}{dp} + \frac{2}{p}x = 12p.$$

Bu tenglamani integrallab, $x = 3p^2 + \dfrac{C}{p^2}$ ni olamiz.

Demak, $x = 3p^2 + \dfrac{C}{p^2}, \quad y = 2p^2 + 2\dfrac{C}{p}, \qquad y = 0$

integral chiziqlar sinfi Lagranj tenglamasini yechimini beradi.

Klero differensial tenglamasi

Ta'rif. ***x*** va ***y*** ga nisbatan chiziqli bo'lgan koeffitsiyentlari esa y' ning funksiyalari bo'lgan quyidagi

$$y = x \cdot y' + \Phi(y')$$

differensial tenglamaga **Klero differensial tenglamasi** deyiladi.

Klero differensial tenglamasi Lagranj differensial tenglamasining xususiy holi hisoblanadi. Ushbu differensial tenglamani yechish algoritmi quyidagicha:

1) $y' = p \;\; \Longrightarrow \;\; y = x \cdot p + \Phi(p)$
2) $y' = p \;\; \Longrightarrow \;\; dy = pdx \;\; \Longrightarrow \;\; dy = d\big(x \cdot p + \Phi(p)\big) \;\; \Longrightarrow$

$$y'dx = pdx + xdp + \Phi'(p)dp \;\; \Longrightarrow \;\; pdx = pdx + xdp + \Phi'(p)dp$$

Oxirgi ifodani *dx* ga bo'lamiz

$$p = p + x\frac{dp}{dx} + \Phi'(p)\frac{dp}{dx} \;\; \Longrightarrow \;\; \big(x + \Phi'(p)\big)\frac{dp}{dx} = 0$$

3) $\begin{cases} x + \Phi'(p) = 0 \\ dp = 0 \end{cases} \;\; \Longrightarrow$

Birinchi yechim: $dp = 0 \;\; \Longrightarrow p = C \;\; \Longrightarrow \;\; y = C \cdot x + \Phi(C).$

Ikkinchi yechim esa: $\begin{cases} y = x \cdot p + \Phi(p) \\ x + \Phi'(p) = 0 \end{cases}$ parametrik tenglamalar sistemasini yechish orqali hosil qilinadi. Hosil bo'lgan *F(x,y)*=0 ikkinchi yechim ixtiyoriy o'zgarmas sonni o'z ichiga olmaydi va umumiy yechimdan ham C ning biror bir qiymati orqali hosil qilinmaydi, demak xususiy yechim emas. Bunday yechimlar **maxsus yechim** (integral) hisoblanadi. Shunday qilib Klero tenglamasining **maxsus yechim**i umumiy yechim (integral) bilan berilgan to'g'ri chiziqlar oilasining **egilish chizigini aniqlaydi**, boshqacha qilib aytganda maxsus yechimning ixtiyoriy nuqtasiga o'tqazilgan urinma ham differensial tenglama yechimi bo'ladi.

Klero differensial tenglamasi ko‘p hollarda analitik geometriyada

2-tartibli egri chiziqlarni qurish uchun ishlatiladi.

Egri chiziqni uning urinmasiga qo‘yilgan xossalari bo‘yicha aniqlaydigan geometrik masalalar Klero tenglamasiga olib keladi. Ushbu xossa aynan urinmaga tegishli bo‘lib, urinadigan nuqtaga tegishli emas. Haqiqatdan ham urinma tenglamasi:

$$Y - y = y'(X - x) \quad yoki \quad Y = y'X + (y - xy')$$

Urinmaning har qanday xossasi $(y - xy')$ va y' o‘rtasidagi munosabat bilan aniqlanadi:

$$F(y - xy', y')=0$$

Ushbu tenglamani $y - xy'$ ga nisbatan yechilsa, aynan

$$y = x \cdot y' + \Phi(y')$$

Klero tenglamasiga kelamiz.

Misol. $y = x \cdot y' + (y')^2$ differensial tenglamani Klero usulida yeching.

1) $y' = p \;\; \Longrightarrow \;\; y = x \cdot p + p^2$

2) $y' = p \;\; \Longrightarrow \;\; dy = pdx \;\; \Longrightarrow \;\; dy = d(x \cdot p + p^2)$

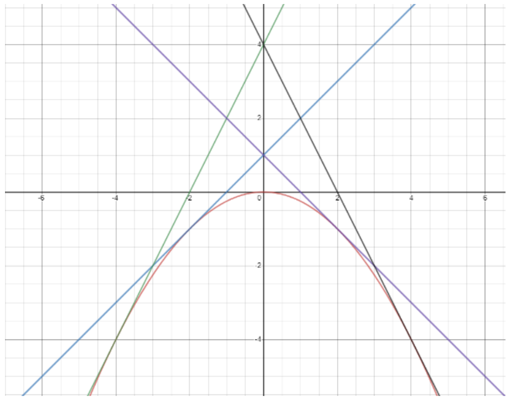
$$y'dx = pdx + xdp + 2pdp \;\; \Longrightarrow \;\; pdx = pdx + xdp + 2pdp$$

Oxirgi ifodani dx ga bo‘lamiz

$p = p + x\frac{dp}{dx} + 2p\frac{dp}{dx} \;\; \Longrightarrow \;\; (x + 2p)\frac{dp}{dx} = 0$ – ushbu tenglama mumkin bo‘lgan ikki xil yechimga ega.

3) $\begin{cases} x + 2p = 0 \\ dp = 0 \end{cases} \Longrightarrow$

1-yechim: $dp = 0 \quad \Longrightarrow p = C \quad \Longrightarrow \quad y = C \cdot x + \varphi(C)$ Klero tenglamasining umumiy integrali (yechimi) to‘g‘ri chiziqlar oilasini tashkil qiladi.



2-yechim: yechim parametrik ko‘rinishda tenglamalar sistemasidan topiladi:

$\begin{cases} y = x \cdot p + p^2 \\ x + 2p = 0 \end{cases} \Rightarrow$

ushbu sistemadan p ni yo‘qotib ikkinchi yechimni topamiz

$$p = -\frac{x}{2} \quad \Rightarrow \quad y = x \cdot \left(-\frac{x}{2}\right) + \left(-\frac{x}{2}\right)^2 = -\frac{x^2}{4} \quad \Rightarrow \quad y = -\frac{x^2}{4}$$

Ikkinchi yechim ixtiyoriy o‘zgarmas sonni o‘z ichiga olmaydi va umumiy yechimdan ham C ning biror bir qiymati orqali hosil qilinmaydi, demak xususiy yechim emas. Bunday yechimlar **maxsus yechim** (integral) hisoblanadi.

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