IKKINCHI TARTIBLI OPERATORLI MATRITSA KO‘RINISHIDAGI UMUMLASHGAN FRIDRIXS MODELINING XOS QIYMATLARI HAQIDA

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Annotatsiya. Ushbu maqolada ikkinchi tartibli operatorli matritsa ko‘rinishidagi umumlashgan Fridrixs modeli $A_\kappa(s)$ ($\kappa > 0$ va $s \in T^d$) qaralgan. $A_\kappa(s)$ operatorli matritsa Fok fazosining nol zarrachali va bir zarrachali qism fazolarining to‘g‘ri yig‘indisida ta’sir qiladi. Maqolada $A_\kappa(s)$ operatorli matritsaning muhim spektrdan tashqarida yotuvchi xos qiymatlarining mavjudlik shartlari va ularning soni tahlil qilingan.

Kalit so‘zlar. Fridrixs modeli, xos qiymat, diskret spektr, muhim spektr.

Abstract. In this paper, a generalized Friedrichs model in the form of a second-order operator matrix $A_\kappa(s)$ ($\kappa > 0$ and $s \in T^d$) is considered. The operator matrix $A_\kappa(s)$ acts in the direct sum of the zero-particle and one-particle subspaces of Fock space. The paper analyzes the conditions for the existence of eigenvalues of the operator matrix $A_\kappa(s)$ lying outside the essential spectrum, as well as their number.

Key words. Friedrichs model, eigenvalue, discrete spectrum, essential spectrum.

Аннотация. В данной статье рассматривается обобщённая модель Фридрихса в виде операторной матрицы второго порядка $A_\kappa(s)$ ($\kappa > 0$ и $s \in T^d$). Операторная матрица $A_\kappa(s)$ действует в прямой сумме нуль-частичного и одночастичного подпространств фоковского пространства. В работе

исследуются условия существования собственных значений операторной матрицы $A_\kappa(s)$, лежащих вне существенного спектра, а также их количество.

Ключевые слова. Модель Фридрихса, собственное значение, дискретный спектр, существенный спектр.

Kirish. Blok operator matritsalar – bu elementlari Banax yoki Hilbert fazolari orasidagi chiziqli operatorlardan iborat bo‘lgan matritsalardir [1]. Blok operator matritsalarning muhim bir sinfi panjaradagi soni saqlanmaydigan sistemalar bilan bog‘liq bo‘lgan Gamiltonianlardir. Bunday masalalar qattiq jism fizikasi [2], kvant maydonlar nazariyasi [3] va statistik fizikada [4, 5] masalalarida yuzaga keladi.

Blok operatorli matritsalarning xos qiymatlarini tadqiq qilish masalasi chiziqli operatorlar nazariyasi hamda matematik fizikaning ko‘plab masalalarida eng faol o‘rganilayotgan obyektlardan biridir. Ushbu maqolada biz ikkinchi tartibli operatorli matritsalar oilasi (umumlashgan Fridrixs modellari deb ataluvchi) $A_\kappa(s)$ ($\kappa > 0$ va $s \in T^d$) ni ko‘rib chiqamiz. Bu operatorli matritsa Fok fazosining nol-zarrachali va bir zarrachali qism fazolarining to‘g‘ri yig‘indisida ta’sir qiladi. Maqolaning asosiy maqsadi ushbu oilaning xossalarini chuqur matematik tahlil qilish hamda tegishli Fredholm determinant yordamida uning muhim spektrdan tashqarida yotuvchi xos qiymatlarini tadqiq qilishdan iborat.

[6-12] ishlarda Fridrixs modeli va umumlashgan Fridrixs modelining muhim va diskret spektrlariga oid tadqiqotlar olib borilgan.

$H_0 \coloneqq \mathbb{C}$ va $H_1 \coloneqq L_2(T^d)$ bo‘lsin. Bu yerda T^d to‘plam d-o‘lchamli tor, ya’ni tomonlari mos ravishda aniqlashtirib biriktirilgan $(-\pi;\pi]^d$ kub. $H \coloneqq H_0 \oplus H_1$ gilbert fazosida quyidagicha aniqlangan umumlashgan Fridrixs modellari oilasi deb ataluvchi $A_\kappa(s)$ ikkinchi tartibli operatorli matritsani aniqlaymiz:

$$A_\kappa(s) := \begin{pmatrix} A_{00} & \kappa A_{01} \\ \kappa A_{01}^* & A_{11}(s) \end{pmatrix}. \tag{1}$$

Uning matritsa elementlari quyidagi tengliklar bilan aniqlangan:

$$A_{oo}f_0 = \tau f_0, \qquad (A_{o1}f_1) = \int_{T^d} f_1(t)\,dt,$$

$$(A_{11}(s)f_1)(x) = \Phi_s(x)\, f_1(x),\ f_i \in \mathcal{H}_i, \qquad i = 0{,}1.$$

Bu yerda τ musbat haqiqiy son, $\Phi_s(\cdot)$ funksiya quyidachida aniqlanadi

$$\Phi_s(x) := \varepsilon\left(\frac{1}{2}(s+x)\right) + \varepsilon(x),$$

$$\varepsilon(x) := \sum_{i=1}^{d}(1 - cosx_i)\,, x = (x_1, x_2, . \ . \ . \ , x_d) \in T^d.$$

Oson ko‘rsatish mumkinki $A_\kappa(s)$ chiziqli, o‘z-o‘ziga qo‘shma va chegaralangan operator bo‘ladi.

$\mathcal{H}_0 \oplus \mathcal{H}_1$ Gilbert fazosida

$$A(s) = \begin{pmatrix} 0 & 0 \\ 0 & A_{11}(s) \end{pmatrix}$$

ikkinchi tartibli operatorli matritsani qaraylik.

U holda $A(s)$ operatorli matritsaning qo‘zg‘alish operatori $A_\kappa(s) - A(s)$ ikki o‘lchamli o‘z-o‘ziga qo‘shma operator bo‘lib, chekli o‘lcjamli qo‘zg‘alishlarda muhim spektrning o‘zgarmasligi haqidagi Veyl teoremasiga ko‘ra $A_\kappa(s)$ va $A(s)$ operatorli matritsalarning muhim spektrlari ustma-ust tushadi.

$A(s)$ operatorli matritsaning aniqlanishiga ko‘ra

$$\sigma\big(A(s)\big) = \sigma_{ess}\big(A(s)\big) = Im\ \Phi_s(x)$$

$\Phi_s(x)$ funksiya T^d torda uzluksiz bo‘lganligi sababli, bu funksiya o‘zining eng katta va eng kichik qiymatlariga erishadi. Ya’ni

$$m(s) \coloneqq \min_{x\epsilon T^d} \Phi_s(x) \quad \text{va} \quad M(s) \coloneqq \min_{x\epsilon T^d} \Phi_s(x)$$

bo’lib, $Im\ \Phi_s(x) = [m(s); M(s)]$ bo‘ladi.

Yuqoridagi mulohazalardan ko‘rsatish mumkinki

$$\sigma_{\text{ess}}\big(A_\kappa(s)\big) = [m(s); M(s)]$$

Har bir fiksirlangan $\kappa > 0$ va $s\epsilon T^d$ uchun quyidagi funksiyani aniqlaymiz.

$$\Delta_\kappa(s;z) = \tau - z - \kappa^2 \int_{T^d} \frac{d\xi}{\Phi_s(\xi) - z}$$

Aniqlanishiga ko‘ra $\Delta_\kappa(s;z)$ funksiya $\mathbb{C}\backslash[m(s);M(s)]$ to‘plamda analitik bo‘ladi. Odatda, bu funksiyaga $A_\kappa(s)$ operatorli matritsaga mos Fredgolm determinant deyiladi.

1-lemma. Har bir $\kappa > 0$ va $s\epsilon T^d$ uchun $z_\kappa(s)\epsilon\mathbb{C}\backslash[m(s);M(s)]$ soni $A_\kappa(s)$ operatorli maatritsaning xos qiymati bo‘lishi uchun $A_\kappa(s;z_\kappa(s)) = 0$ bo‘lishi zarur va yetarli.

1-teorema. Har bir tayin $\kappa > 0$ va $s\epsilon \mathrm{T}^d$ da $A_\kappa(s)$ operatorli matritsa $(-\infty{:}\, m(s))$ oraliqda ko‘pi bilan bitta xos qiymatga ega.

Isbot. $A_\kappa(s)$ operatorli matritsaning muhim spektr $\sigma_{ess}\big(A_\kappa(s)\big) = [m(s);M(s)]$ dan tashqarida yotuvchi xos qiymatlarini tahlil qilish uchun unga mos $\Delta_\kappa(s;z)$ Fredgolm determinantining nollarini o‘rganamiz.

Dastlab $\Delta_\kappa(s;z)$ funksiyani monotonlikka tekshiramiz:

$$\frac{\partial \Delta_\kappa(s;z)}{\partial z} = -1 - \kappa^2 \int_{\mathrm{T}^d} \frac{d\xi}{(\Phi_s(\xi) - z)^2}$$

munosabatga e’tibor beradigan bo‘lsak, $\forall z\epsilon\mathbb{R}\backslash[m(s);M(s)]$ uchun $\frac{\partial \Delta_\kappa(s;z)}{\partial z} < 0$ bo‘ladi. Demak, $\Delta_\kappa(s;z)$ funksiya monoton kamayuvchi funksiya bo‘ladi.

Ikkinchi tomondan

$$\lim_{z\to-\infty} \Delta_\kappa(s;z) = +\infty \qquad (2)$$

bo‘ladi.

1) Faraz qilaylik $\int_{\mathrm{T}^d} \frac{d\xi}{\Phi_s(\xi) - m(s)}$ integral uzoqlashuvchi bo‘lsin. U holda $\Delta_\kappa(s;m(s)) = -\infty$ bo‘lib, $\Delta_\kappa(s;z)$ funksiyaning $(-\infty;m(s))$ oraliqda uzluksiz va

monoton kamayuvchi ekanligi hamda (2) munosabatni inobatga olsak, u holda shunday yagona $z_0 \epsilon (-\infty; m(s))$ soni topilib, $\Delta_\kappa(s; z_0) = 0$ bo‘ladi. 1-lemmaga ko‘ra z_0 soni $A_\kappa(s)$ operatorli matritsaning xos qiymati bo‘ladi.

Demak, bu holatda $A_\kappa(s)$ operatorli matritsa $(-\infty; m(s))$ oraliqda 1 ta xos qiymatga ega.

2) $\int_{\mathrm{T}^d} \frac{d\xi}{\Phi_s(\xi) - m(s)}$ integral yaqinlashuvchi bo‘lib, uning qiymati $I(m(s))$ ga teng bo‘lsin.

2.1) Agar $\tau < m(s)$ bo‘lsa, u holda ixtiyoriy $\kappa > 0$ uchun $\Delta_\kappa(s; m(s)) < 0$ bo‘lib, $\Delta_\kappa(s; z)$ funksiyaning $(-\infty; m(s))$ oraliqda uzluksiz, monoton kamayuvchi ekanligi hamda (2) munosabatga ko‘ra bu funksiya $(-\infty; m(s))$ oraqliqda yagona z_0 ega bo‘ladi. 1-lemmaga ko‘ra $\Delta_\kappa(s)$ funksiyaning noli bo‘lgan z_0 soni $A_\kappa(s)$ operatorli matritsaning xos qiymati bo‘ladi. Demak, bu holda $A_\kappa(s)$ operatorli matritsa $(-\infty; m(s))$ oraliqda yagona xos qiymatga ega.

2.2) Faraz qilaylik $\tau > m(s)$ bo‘lib, $\kappa > \sqrt{\tau - m(s)}(I(m(s)))^{-\frac{1}{2}}$ bo‘lsin. U holda ko‘rsatish mumkinki, $\Delta_\kappa(s; m(\kappa)) < 0$ bo‘ladi. Yana $\Delta_\kappa(s; z)$ funksiyaning $(-\infty; m(s))$ oraliqda uzluksiz, monoton kamayuvchiligi hamda (2) munosabatdan foydalansak $\Delta_\kappa(s; z)$ funksiya $(-\infty; m(s))$ oraliqda yagona nolga ega degan xulosaga kelamiz.

1 -lemmadan esa $\Delta_\kappa(s; z)$ ning noli bo‘lgan bu son $A_\kappa(s)$ operatorli matritsaning xos qiymati bo‘lishi kelib chiqadi. Demak, bu holda ham $A_\kappa(s)$ operatorli matritsa $m(s)$ dan chapda yagona xos qiymatga ega bo‘ladi.

2.3) Faraz qilaylik $\tau > m(s)$ bo‘lib, $\kappa < \sqrt{\tau - m(s)}(I(m(s)))^{-\frac{1}{2}}$ bo‘lsin. U holda $\Delta_\kappa(s; m(\kappa)) > 0$ bo‘ladi. Yuqoridagi kabi mulohaza yuritadigan bo‘lsak, $\Delta_\kappa(s; z)$ funksiya $(-\infty; m(s))$ oraliqda nolga ega emasligi kelib chiqadi. Ya’ni bu holda $A_\kappa(s)$ operatorli matritsa $(-\infty; m(s))$ oraliqda xos qiymatga ega emas degan xulosa qilish mumkin.

Quyidagi teorema xuddi 1-teorema kabi isbotlanadi.

2-teorema. Har bir tayin $\kappa > 0$ va $s \epsilon \mathrm{T}^d$ uchun $A_\kappa(s)$ operatorli matritsa $(M(s); +\infty)$ oraliqda ko‘pi bilan bitta xos qiymatga ega.

Yuqoridagi 1- va 2- teoremalardan quyidagicha xulosa chiqarish mumkin:

$A_\kappa(s)$ operatorli matritsa muhim spektrdan tashqarida 2 ta xos qiymatga ega bo‘lsa, bu xos qiymatlardan bittasi muhim spektrdan chapda, ikkinchisi esa muhim spektrdan o‘ngda yotadi.

Xulosa. Ushbu maqolada Fok fazosining qirqilgan nol-zarrachali va bir-zarrachali qismfazolarining to‘g‘ri yig‘indisida ta’sir qiluvchi $A_\kappa(s)$ operatorli matritsaning ba’zi xossalari ko‘rib chiqildi. Uning muhim spektrdan tashqarida yotuvchi xos qiymatlari mavjud bo‘lish hsartlari va ularning soni to‘liq tahlil qilindi. Buning uchun $A_\kappa(s)$ operatorli matritsaga mos Fredgolm determinant qurilib uning nollari yordamida operatorli matritsaning xos qiymatlari tahlil qilindi.

FOYDALANILGAN ADABIYOTLAR RO‘YXATI

1. Tretter C. Spectral theory of block operator matrices and applications. London. Imperial College Press. 2008. p.264

2. A. I. Mogilner, *Hamiltonians in solid state physics as multiparticle discrete Schrodinger operators: problems and results*, Advances in Sov. Math. **5** (1991), 139-194.

3. K. O. Friedrichs, *Perturbation of Spectra in Hilbert Space*, American Mathematical Society, Providence, RI, 1965.

4. V. A. Malishev, R. A. Minlos, *Linear Infinite-Particle Operators*, Transl. Math. Monographs, Vol. 143, American Mathematical Society, Providence, RI, 1995.

5. R. A. Minlos, H. Spohn, *The three-body problem in radioactive decay: the case of one atom and at most two photons*. Topics in Statistical and Theoretical

Physics. Amer. Math. Soc. Transl. Ser. 2, **177**, American Mathematical Society, Providence, RI, 1996, 159-193.

6. Расулов Т.Х., Бахронов Б.И. О спектре тензорной суммы моделей Фридрихса. *Молодой учёный*, 9 (2015), 17-20.

7. Bahronov B.I., Rasulov T.H. Structure of the numerical range of Friedrichs model with rank two perturbation. *European science*, 2-2(51) (2020), 15-18.

8. Rasulov T.H., Dilmurodov E.B. Eigenvalues and virtual levels of a family of 2x2 operator matrices. *Methods Func. Anal. Topology*, 1(25) (2020), 273-281.

9. Rasulov T.H., Dilmurodov E.B. Threshold analysis for a family of 2x2 operator matrices. *Nanosystems: Phys., Chem., Math.*, 6(10) (2019), 616-622.

10. Расулов Т.Х., Дилмуродов Э.Б. Исследование числовой области значений одной операторной матрицы. *Вестн. Самарск. госуд. техн. ун-та. Сер. Физ.-мат. науки.*, 35 (2) (2014), 50–63.

11. Расулов Т.Х., Дилмуродов Э.Б. Бесконечность числа собственных значений операторных (2x2)-матриц. Асимптотика дискретного спектра. *ТМФ*. 3(205) (2020), 368-390.

12. Dilmurodov E. (2020). Discrete eigenvalues of a 2x2 operator matrix. *ArXiv:2011.09650*. 1-12.