ANALYSIS OF THE PROBLEMS OF MAXIMIZING PRODUCTION VOLUME IN THE CONDITIONS OF LIMITED BUDGET IN THE ACTIVITIES OF ENTERPRISES USING MATHEMATICAL METHODS

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Annotation: This scientific article analyzes the problems of maximizing production volume in the conditions of limited budget in the activities of enterprises using mathematical methods. The relevance of the research is that in a modern competitive market, the allocation of resources (labor and capital) based on empirical or intuitive factors leads to economic inefficiency for enterprises. The Cobb-Douglas production function and the Lagrange multiplier method (conditional extremum) were used as a solution to the problem. The mathematical calculations and the results obtained show that the optimal proportion of resources is directly related to their prices and the level of elasticity in production. The scientific and practical contribution of the study is that a specific mathematical algorithm and the optimal point for the distribution of limited financial resources between capital and labor were calculated on the example of a local textile enterprise.

Keywords: production function, Cobb-Douglas model, optimization, Lagrangian method, labor and capital, budget constraint, marginal productivity, economic and mathematical modeling.

INTRODUCTION

The main problem of any economic system is to satisfy unlimited needs with limited resources. At the microeconomic level, this problem directly faces manufacturing enterprises: how to achieve the highest production volume (and, accordingly, high profits) at the expense of limited budget funds available? A clear and reliable answer to this question can be found only through economic and mathematical modeling.

The problem raised in the article is to create mechanisms for purchasing and distributing resources (in particular, labor and equipment/capital) not randomly, but based on strict mathematical laws. The relevance of the study is that at a time when foreign and domestic investments are accelerating in the economy of Uzbekistan, enterprise managers must make strategic decisions on directing funds to new technologies (capital) or creating additional jobs (labor). If the marginal productivity and prices of factors are not mathematically analyzed in this process, the enterprise will not be able to fully utilize its potential. The scientific novelty and purpose of this article is to solve the Cobb-Douglas modification for an enterprise with a limited financial budget through the Lagrangian function and numerically prove the absolute optimal volumes of resources.

LITERATURE REVIEW

The most important discovery in the mathematical representation of production processes in economics is the production function, created in 1928 by American scientists - mathematician Charles Cobb and economist Paul Douglas. They, having practically analyzed the US manufacturing industry, empirically proved that the volume of output is to a certain extent dependent on the amount of labor and capital. Later, V. Leontiev developed constant proportional functions, and scientists such as K. Arrow and R. Solow developed CES (constant elasticity of substitution) functions.

The work of D. Hicks and P. Samuelson on the mathematical solution of production optimization problems is particularly noteworthy. They actively applied the theory of differential calculus and conditional extremum (Lagrange multipliers) to economics. Among our domestic economists, T.Sh. Shodiev, B.Y. Khodiev and F.O. Khusanov's textbooks and training manuals systematically cover such areas as linear and nonlinear programming, resource allocation, and inter-sectoral balance models on the example of Uzbek enterprises. However, the mechanisms for practical application of this theoretical knowledge within the framework of the local budget of a specific enterprise and current market prices constantly require new scientific and practical research.

RESEARCH METHODOLOGY

In this study, the Cobb-Douglas production function was used to model the production volume of the enterprise. Its mathematical form is as follows:

$$Q = A \cdot L^{\alpha} \cdot K^{\beta}$$

Q – volume of output;

A – coefficient of technological progress (scale parameter, □(A>0));

L – amount of labor expended (for example, number of workers or hours worked);

K – amount of capital used (number of equipment, machines);

α and β – coefficients of elasticity of labor and capital, respectively, in terms of production (indicate by how much percent output increases when the factors increase by 1%).

The enterprise cannot increase production indefinitely because its financial resources (budget) are limited. The budget constraint equation (Isocost) is written as:

$$C = P_L \cdot L + P_K \cdot K$$

Here C is the current budget amount; $P_L -$ is the cost of labor (wage rate); $P_K -$ is the cost of capital (equipment rental or depreciation cost). The mathematical problem before us is to find the maximum value of the function Q when the condition $C = P_L L + P_K K$ is met. To solve this problem, the Lagrange multiplier method (the method of finding the conditional extremum) is used. The Lagrange function is constructed as follows:

$$\mathcal{L}(L, K, \lambda) = A \cdot L^{\alpha} \cdot K^{\beta} + \lambda(C - P_L L - P_K K)$$

Here λ is the Lagrange multiplier (a hidden price that indicates how much production volume increases when the budget increases by 1 unit). To find the optimal volumes, the partial derivatives of the function $\mathcal{L}$ with respect to L, K and λ are taken and set to zero:

$$\frac{\partial \mathcal{L}}{\partial L} = \alpha A L^{\alpha-1} K^{\beta} - \lambda P_L = 0$$

$$\frac{\partial \mathcal{L}}{\partial K} = \beta A L^{\alpha} K^{\beta-1} - \lambda P_K = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = C - P_L L - P_K K = 0$$

From this system of equations, it follows that the basic law of equilibrium is that the ratio of the marginal (marginal) products of factors must be equal to the ratio of their prices.

DISCUSSION AND RESULTS

In order to demonstrate the practical significance of the mathematical apparatus presented in the methodology, we conduct an analysis based on data from the textile enterprise "Textile-Pro" LLC.

The monthly investment budget of the enterprise is $C = 1200$ million soums. The monthly cost of one work team in the market is $P_L = 30$ million soums, and the monthly cost of one sewing machine complex is $P_K = 40$ million soums. As a result of econometric analysis, it was determined that the production function of the enterprise is of the form $Q = 10 \cdot L^{0.6} \cdot K^{0.4}$ (where $A = 10$, $\alpha = 0.6$, $\beta = 0.4$).

The goal is to distribute the budget of 1200 million soums between L and K in such a way that the result is Q as large as possible. According to the optimality condition (from the ratio of specific derivatives):

$$\frac{\alpha \cdot K}{\beta \cdot L} = \frac{P_L}{P_K} \Longrightarrow \frac{0.6 \cdot K}{0.4 \cdot L} = \frac{30}{40}$$

Let's simplify the equation:

$$1.5 \cdot \frac{K}{L} = 0.75 \Longrightarrow K = 0.5 \cdot L$$

We substitute the resulting equation $K = 0.5L$ into the budget constraint equation ($30L + 40K = 1200$):

$$30L + 40(0.5L) = 1200$$

$$30L + 20L = 1200 \Longrightarrow 50L = 1200 \Longrightarrow L^{*} = 24$$

Once the amount of labor (L^*) is determined, we find the amount of capital:

$$K^* = 0.5 \cdot 24 = 12$$

Therefore, the optimal strategy for the enterprise is to hire a crew of 24 workers and 12 sets of equipment. We calculate the maximum volume of output that can be produced at these optimal values:

$$Q_{max} = 10 \cdot (24)^{0.6} \cdot (12)^{0.4} \approx 10 \cdot 6.74 \cdot 2.70 \approx 182 \,.$$

To show the analytical comparison of the results, we consider other possible scenarios in the table:

Table 1

VARIOUS OPTIONS OF BUDGET DISTRIBUTION AND THEIR MATHEMATICAL RESULTS [1]

Strategy options	**Labor (L)**	**Capital (K)**	**Budget cost (30L+40K)**	**Production capacity (Q)**
Option 1 (Emphasis on work only)	32	6	1200 million	≈ 163 unity
Option 2 (Mathematically optimal)	**24**	**12**	**1200 million**	**≈ 182 unity**
Option 3 (Emphasis on capital letters only)	16	18	1200 million	≈ 168 unity

As can be seen from the table, if managers spend the funds in equal halves (or voluntarily) without mathematical calculations, the production volume will be much lower than the optimal value (163 or 168 units). It is at the points □(L=24) and □(K=12) found using the Lagrange method that the enterprise achieves the highest result with 182 units.

CONCLUSIONS AND SUGGESTIONS

Based on the studies on the optimization of production functions using the Cobb-Douglas model and differential calculus (Lagrange method), the following important conclusions and practical suggestions were formulated:

Firstly, the optimal proportion of resources in any enterprise is strictly dependent not only on their market price, but also on the elasticity of productivity (□α and □β parameters) in this particular enterprise. If the enterprise operates in a labor-intensive industry (□(α>β)), even if it is cheap, the money invested in technology will not be as effective as the money invested in labor.

Secondly, business entities are recommended to recalculate the parameters □(A,α,β) using econometric methods on a quarterly basis based on their empirical data (costs and output volume). This is because technological changes and fluctuations in market prices (□(P_L,P_K)) constantly shift the optimal point.

Thirdly, the era of "estimating by eye" production processes is over. In a highly competitive economy, the widespread use of economic and mathematical models and nonlinear programming tools, which provide absolute accuracy in making management decisions and business plans, is the main shield against enterprise bankruptcy.

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