KO'P O'ZGARUVCHILI FUNKSIYA UCHUN SILVESTR ALOMATINING BA'ZI TADBIQLARI

Jumaboyeva Gulhayo Baxtiyor qizi

Buxoro davlat universiteti magistri

Annotasiya. *Ushbu maqolada ko'p o'zgaruvchili funsiya uchun Silvestr alomatining ba'zi tadbiqlarining isbotlari keltirilgan. Silvestr alomatining amaliy tadbiqlari sifatida bir nechta misollar yechib ko'rsatilgan.*

Kalit so'zlar: *Silvestr alomati, minimum, maksimum, uzluksizlik, xususiy hosila, kvadratik forma, koeffitsent, ekstremumning yetarli sharti.*

Abstract. This paper provides rigorous proofs of several applications of Sylvester's criterion in the context of multivariable functions. To illustrate its practical utility, a number of representative examples are solved and discussed.

Keywords: Sylvester's criterion, minimum, maximum, continuity, partial derivative, quadratic form, coefficient, sufficient condition for an extremum.

Аннотация. В данной статье приведены строгие доказательства ряда приложений критерия Сильвестра в контексте функций многих переменных. Для демонстрации его практической полезности рассмотрены и решены несколько показательных примеров.

Ключевые слова:как критерий Сильвестра, минимум, максимум, непрерывность, частные производные, квадратичные формы, коэффициенты, а также достаточные условия существования экстремума.

1-ta'rif. n ta $x_1, x_2, \dots\dots.., x_n$ noma'lumlarning $f(x)$ kvadratik formasi deb har bir hadi bu no'malumlarning kvadrati yoki ikkita noma'lumning ko'paytmasidan iborat bo'lgan

$$f = \sum_{i=1}^{n} \sum_{i=1}^{n} a_{ij} x_i x_j \tag{1}$$

yig'indiga aytiladi.

Kvadratik formaning a_{ij} koeffitsiyentlaridan

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots\dots & a_{1n} \\ a_{21} & a_{22} & \dots\dots & a_{2n} \\ \dots & \dots & \dots\dots & \dots \\ a_{n1} & a_{n2} & \dots\dots & a_{nn} \end{pmatrix}$$

kvadrat matritsani tuzish mumkin. Bu yerda A matritsaning barcha xarakteristik ildizlari haqiqiy bo'lishi uchun $a_{ij} = a_{ji}$ deb faraz qilinadi. A matritsaning rangi (1) kvadratik formaning rangi deyiladi. A matritsa aynimagan bo'lsa, (1) kvadratik forma xosmas deyiladi.

Kvadratik formaning koeffitsiyentlari haqiqiy yoki kompleks sonlar bo'lishiga bog'liq holda, kvadratik forma haqiqiy yoki kompleks deyiladi.

Bizga ushbu

$$P(\xi_1, \xi_2, \xi_3, \dots\dots\dots\dots, \xi_m) = \sum_{i,k=1}^{m} a_{ik}\xi_1 \cdot \xi_k$$

Kvadratik forma berilgan bo'lsin va uning musbat aniqlangan bo'lishi uchun

$$a_{11} > 0, \ \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} > 0, \ \begin{vmatrix} a_{11} & a_{12} & \dots\dots & a_{1n} \\ a_{21} & a_{22} & \dots\dots & a_{2n} \\ \dots & \dots & \dots\dots & \dots \\ a_{m1} & a_{m2} & \dots\dots & a_{mn} \end{vmatrix} > 0$$

tengsizliklarning, manfiy aniqlangan bo'lishi uchun esa

$$a_{11} < 0, \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} > 0, \dots\dots, \qquad (-1)^m \begin{vmatrix} a_{11} & a_{12} & \dots\dots & a_{1n} \\ a_{21} & a_{22} & \dots\dots & a_{2n} \\ \dots & \dots & \dots\dots & \dots \\ a_{m1} & a_{m2} & \dots\dots & a_{mn} \end{vmatrix} > 0$$

tengsizliklarning bajarilishi zarur va yetarli.

Xususiy holni funksiya ikki o'zgaruvchiga bog'liq bo'lgan holni qaraylik. $f(x_1, x_2)$ funksiya $x^0 = (x_1^0, x_2^0)$ nuqtaning biror atrofi

$$U_\delta(x^0) = \{x = (x_1, x_2) \in R^2: \ \rho(x, x^0) < \delta\} \ (\delta > 0)$$

da birinchi, ikkinchi tartibli uzluksiz hosilalarga ega bo'lib, x^0 esa qaralayotgan funksiyaning statsionar nuqtasi bo'lsin:

$$f'_{x_1}(x^0) = 0, \qquad f''_{x_2}(x^0) = 0,$$

Odatdagidek, $a_{11} = f'_{x_1{}^2}(x^0)$, $a_{12} = f''_{x_1x_2}(x^0), a_{22} = f'_{x_2{}^2}(x^0)$

1) Agar

   $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}^2 > 0$ va $a_{11} > 0$ bo'lsa, $f(x)$ funksiya x^0 nuqtada minimumga erishadi.

2) Agar

   $a_{11}a_{22} - a_{12}^2 > 0$ va $a_{11} < 0$

   bo'lsa, $f(x)$ funksiya x^0 nuqtada maksimumga erishadi.

3) Agar $a_{11}a_{22} - a_{12}^2 < 0$ bo'lsa, $f(x)$ funksiya x^0 nuqtada ekstremumga erishmaydi.

4) Agar $a_{11}a_{22} - a_{12}^2 = 0$ bo'lsa, $f(x)$ funksiya x^0 nuqtada ekstremumga erishishi mumkin, erishmasligi ham mumkin.

Haqiqatan ham 1) va 2)- hollarda kvadratik forma mos ravishda musbat aniqlangan yoki manfiy aniqlangan bo'ladi. (Silvestor alomati)

3)- holda, ya'ni

$$a_{11}a_{22} - a_{12}^2 < 0 \qquad (2)$$

bo'lganda $Q(\xi_1, \xi_2) = a_{11}\xi_1^2 + 2a_{12}\xi_1\xi_2 + a_{22}\xi_2^2$ kvadratik forma noaniq bo'ladi. Shuni isbotlaylik.

$a_{11} = 0$ bo'lsin. Bu holda (2) dan $a_{12} \neq 0$ bo'lishi kelib chiqadi. Natijada $Q(\xi_1, \xi_2)$ kvadratik forma ushbu

$$Q(\xi_1, \xi_2) = (2a_{12}\xi_1 + a_{22}\xi_2)\xi_2$$

ko'rinishga keladi. Bu kvadratik forma

$$\xi_1 = \frac{1 - a_{22}}{2a_{12}}, \quad \xi_2 = 1$$

qiymatda musbat:

$$Q\left(\frac{1-a_{22}}{2a_{12}}, 1\right) = 1 > 0 \ \ va \ \xi_1 = \frac{1+a_{22}}{2a_{12}} \ \ \xi_2 = -1$$

Qiymatda esa manfiy:

$$Q\left(\frac{1+a_{22}}{2a_{12}},-1\right) = -1 < 0$$

bo'ladi.

Shunday qilib, $a_{11}a_{22} - a_{12}^2 < 0$ bo'lganda $Q(\xi_1, \xi_2)$ kvadratik formaning noaniq bo'lishi isbot etildi.

4) – holni, ya'ni $a_{11}a_{22} - a_{12}^2 = 0$ bo'lgan holni qaraylik. Bu holda $a_{11} = 0$ bo'lsa, unda $a_{12} = 0$ bo'lib, $Q(\xi_1, \xi_2)$ kvadratik forma ushbu

$$Q(\xi_1, \xi_2) = a_{22}\xi_2^2$$

ko'rinishini oladi.

Ravshanki, $a_{22} > 0$ bo'lganda $Q(\xi_1, \xi_2) \geq 0$,

$a_{22} < 0$ bo'lganda $Q(\xi_1, \xi_2) \leq 0$

bo'lib, ξ_1 ning ixtiyoriy qiymatida $Q(\xi_1, 0) = 0$ bo'ladi.

Agar $a_{11} > 0$ bo'lsa, $Q(\xi_1, \xi_2) = a_{11}(\xi_1 + \frac{a_{12}}{a_{11}}\xi_2)^2 \leq 0$,

$a_{11} < 0$ bo'lganda $Q(\xi_1, \xi_2) = a_{11}(\xi_1 + \frac{a_{12}}{a_{11}}\xi_2)^2 \leq 0$,

bo'lib, ξ_1 va ξ_2 larning

$$\xi_1 = -\frac{a_{12}}{a_{11}}\xi_2$$

tenglikni qanoatlantiruvchi barcha qiymatlarida $Q(\xi_1, \xi_2)$ kvadratik forma nolga teng bo'ladi. Demak, qaralayotgan holda $Q(\xi_1, \xi_2)$ kvadratik forma yarimmusbat aniqlangan yoki yarimmanfiy aniqlangan bo'ladi.

$$u_1(x) + u_1(y) = \sum_{i=1}^{3}(1 - cosx_i) +$$

$$+\sum_{i=1}^{3}(1 - cosy_i) = 6 - cosx_1 - cosx_2 - cosx_3 - cosy_1 - cosy_2 - cosy_3$$

$$\frac{\partial u_1(x)}{\partial x_1} = sinx_1, \qquad \frac{\partial {u_1}^2(x)}{\partial x_1 \partial x_2} = 0 \quad \frac{\partial u_1(x)}{\partial x_1 \partial x_1} = cosx_1$$

$$\frac{\partial u_1(x)}{\partial x_i \partial x_j} = \begin{cases} cosx_i, & i = j \\ 0 & i \neq j \end{cases}$$

$$\frac{\partial u_1(x)}{\partial y_i \partial y_j} = \begin{cases} cosy_i\,, & i = j \\ 0 & i \neq j \end{cases}$$

$$A = \begin{pmatrix} \frac{\partial u^2(x)}{\partial x_1 \partial x_1} & \frac{\partial u^2(x)}{\partial x_1 \partial x_2} & \frac{\partial u^2(x)}{\partial x_1 \partial x_3} & \frac{\partial u^2(x)}{\partial x_1 \partial y_1} & \frac{\partial u^2(x)}{\partial x_1 \partial y_2} & \frac{\partial u^2(x)}{\partial x_1 \partial y_3} \\ \frac{\partial u^2(x)}{\partial x_2 \partial x_1} & \frac{\partial u^2(x)}{\partial x_2 \partial x_2} & \frac{\partial u^2(x)}{\partial x_2 \partial x_3} & \frac{\partial u^2(x)}{\partial x_2 \partial y_1} & \frac{\partial u^2(x)}{\partial x_2 \partial y_2} & \frac{\partial u^2(x)}{\partial x_2 \partial y_3} \\ \frac{\partial u^2(x)}{\partial x_3 \partial x_1} & \frac{\partial u^2(x)}{\partial x_3 \partial x_2} & \frac{\partial u^2(x)}{\partial x_3 \partial x_3} & \frac{\partial u^2(x)}{\partial x_3 \partial y_1} & \frac{\partial u^2(x)}{\partial x_3 \partial y_2} & \frac{\partial u^2(x)}{\partial x_3 \partial y_3} \\ \frac{\partial u^2(x)}{\partial y_1 \partial x_1} & \frac{\partial u^2(x)}{\partial y_1 \partial x_2} & \frac{\partial u^2(x)}{\partial y_1 \partial x_3} & \frac{\partial u^2(x)}{\partial y_1 \partial y_1} & \frac{\partial u^2(x)}{\partial y_1 \partial y_2} & \frac{\partial u^2(x)}{\partial y_1 \partial y_3} \\ \frac{\partial u^2(x)}{\partial y_2 \partial x_1} & \frac{\partial u^2(x)}{\partial y_2 \partial x_2} & \frac{\partial u^2(x)}{\partial y_2 \partial x_3} & \frac{\partial u^2(x)}{\partial y_2 \partial y_1} & \frac{\partial u^2(x)}{\partial y_2 \partial y_2} & \frac{\partial u^2(x)}{\partial y_2 \partial y_3} \\ \frac{\partial u^2(x)}{\partial y_3 \partial x_1} & \frac{\partial u^2(x)}{\partial y_3 \partial x_2} & \frac{\partial u^2(x)}{\partial y_3 \partial x_3} & \frac{\partial u^2(x)}{\partial y_3 \partial y_1} & \frac{\partial u^2(x)}{\partial y_3 \partial y_2} & \frac{\partial u^2(x)}{\partial y_3 \partial y_3} \end{pmatrix}$$

$$A = \begin{pmatrix} cosx_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & cosx_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & cosx_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & cosy_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & cosy_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & cosy_3 \end{pmatrix}$$

I hol. $x = (0,0,0)$ va $y = (0,0,0)$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_1 = 1$$

$$A_2 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 > 0 \quad A_3 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 > 0$$

$$A_4 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 1 > 0$$

$$A_5 = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} = 1 > 0$$

$$A_6 = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} = 1 > 0$$

A matritsaning barcha bosh minorlarining determinantlari musbat bo'lganligi uchun musbat aniqlangan bo'ladi.

II hol. $x = (\pi, \pi, \pi), y = (\pi, \pi, \pi)$

$$A = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$A_1 = -1 < 0$$

$$A_2 = \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} = 1 > 0 \quad A_3 = \begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = -1 < 0$$

$$A_4 = \begin{vmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} = 1 > 0$$

$$A_5 = \begin{vmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{vmatrix} = -1 < 0$$

$$A_6 = \begin{vmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{vmatrix} = 1 > 0$$

Bunda bosh minorlar ning barchasi musbat emas. Demak bu matritsa musbat aniqlanmagan.

III hol. $x = (0, \pi, 0) \quad y = (0, \pi, 0)$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$A_1 = 1$

$A_2 = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = -1 < 0 \quad A_3 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -1 < 0$

$$A_4 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = -1 < 0$$

$$A_5 = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} = 1 > 0$$

$$A_6 = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} = 1 > 0$$

Bu holda ham matritsa musbat aniqlanmagan.

IV hol. $x = (0, \pi, 0) \quad y = (\pi, 0{,}0)$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$A_1 = 1$

$A_2 = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = -1 < 0 \quad A_3 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -1 < 0$

$$A_4 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} = 1 > 0$$

$$A_5 = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} = 1 > 0$$

$$A_6 = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} = 1 > 0$$

Bu holda ham matritsa musbat aniqlanmagan.

Xulosa: *Ushbu maqolada ko‘p o‘zgaruvchili funsiya uchun Silvestr alomatining ba’zi tadbiqlarining isbotlari keltirilgan hamda Silvestr alomatining amaliy tadbiqlari sifatida bir nechta misollar yechib ko‘rsatilgan.*

Foydalanilgan adabiyotlar

1. Xudayberganov G., Vorisov A. K., Mansurov X. T., Shoimqulov B. A. Matematik analizdan ma’ruzalar, I, II q. T. «Voris-nashriyot», 2010.

2.Кудрявцев Л.Д. Математический анализ. т.2, М.: Высшая школа, 1973.

3.Фихтенгольц Г.М. Курс дифференциального и интегрального исчисления. Т. 3, М.: Наука, 1969.

4.Tosheva N. A., Ismoilova D. E. Ikki kanalli molekulyar-rezonans modelining rezolventasi //Scientific progress. – 2021. – Т. 2. – №. 2. – С. 580-586.

5.Тошева, Наргиза Ахмедовна. "О ветвях существенного спектра одной 3x3-операторной матрицы." Наука, техника и образование 2-2 (2021): 44-47.

6.Tosheva N. A., Ismoilova D. E. Ikki kanalli molekulyar-rezonans modelining sonli tasviri //Scientific progress. – 2021. – Т. 2. – №. 1. – С. 1421-1428.

7.Rasulov T., Tosheva N. Main property of regularized Fredholm determinant corresponding to a family of 3× 3 operator matrices //European science. – 2020. – Т. 2. – С. 51.

8.Tosheva N. A., Ismoilova D. E. The presence of specific values of the two-channel molecular-resonance model //Scientific progress. – 2021. – Т. 2. – №. 1. – С. 111-120.