STUDY OF THE STRESSED-DEFORMED STATE OF AN ELASTIC HALF-SPACE IN CONTACT PROBLEMS

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Abstract: In contact problems, the influence dimensions of elastic bodies are of great importance. This allows for the simplification of boundary conditions and the application of mathematical methods of elasticity theory.

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In contact problems of the theory of elasticity, the interaction of elastic bodies whose external geometry does not correspond to each other is widely studied. In such problems, the dimensions of the interaction surface are assumed to be very small compared to the total dimensions of the bodies.

Since the dimensions of the bodies are much larger than the dimensions of the interaction surface, the stresses generated in the interaction zone practically do not depend on their external configuration at points located sufficiently far from the boundaries of the body. Therefore, one of the interacting bodies can be considered as a semi-infinite elastic medium.

This formulation of the problem significantly simplifies the boundary conditions and makes it possible to effectively use the mathematical methods of the theory of elasticity. Therefore, this approach is widely used in solving contact problems.

Below, one of the cases presented in the literature, is considered - the state of stress under the influence of external stresses applied along a path of finite width and sufficiently large length of an elastic half-space (Fig. 1).

In the introduced coordinate system, the *0xy* plane coincides with the boundary of the elastic half-space, and the 0z axis is directed into the half-space. The width of the stress space is *a*+*b*, and it is directed along the 0y axis. In addition, the half-elastic space is considered to be in a state of uniform deformation, i.e. $\varepsilon_y = 0$.

Thus, the transverse section of the semi-elastic space is affected by the normal stresses p(x) and the shear stresses q(x) for the part *z=0*, $-b \le x \le a$. It is required to determine the stresses $\sigma_x, \sigma_z, \sigma_{xz}$, the deformation tensor components, and the displacement components U_x, U_z at the internal points of the transverse section.

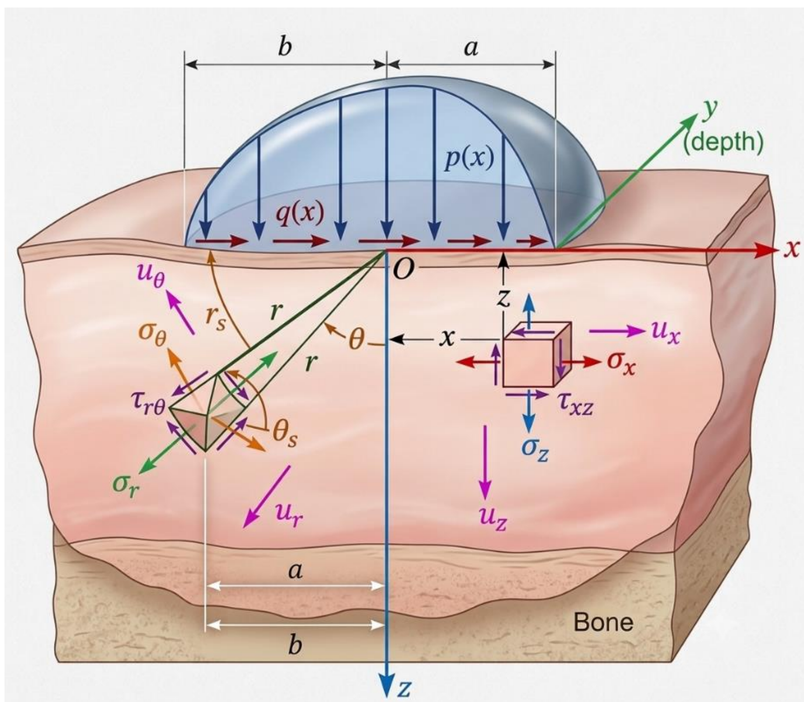

Fig.-1

As noted, such an issue is borderline

$$\begin{cases} x < -b \\ x > a \end{cases}, \quad \overline{\sigma_z} = \overline{\tau_{xy}} = 0,$$

and $-b \le x \le a$ is biharmonic satisfying the conditions in the sphere $\overline{\sigma_z} = -p(x)$, $\overline{\tau_{xz}} = -q(x)$,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2}\right) = 0, \qquad (1)$$

satisfying $\varphi(x,z)$

$$\sigma_z = \frac{\partial^2 \varphi}{\partial x^2}, \qquad \sigma_x = \frac{\partial^2 \varphi}{\partial z^2}, \qquad \tau_{xy} = -\frac{\partial^2 \varphi}{\partial x \partial y}, \tag{2}$$

is brought to define the function. Also, based on the problem statement, sufficiently far from the "contact zone" $(x \to \infty, z \to 0)$ stresses are sufficient to satisfy conditions $\sigma_x \to 0, \sigma_z \to 0, \tau_{xy} \to 0$.

To solve specific problems, in most cases (especially in stamp problems), instead of boundary stresses, displacements $\overline{U_x}$ and $\overline{U_x}\,(x)$ are given based on the stamp geometry. If there is no slip under the action of the stamp,

$$q(x) = \pm \mu p(x),$$

(μ - the coefficient of friction in sliding) and in the case of sliding, $q(x) = 0$ conditions are taken into account.

In the case of stamping problems, depending on the formulation of the problem and its nature, the following boundary conditions are imposed:

In the general theory presented above, we have considered the state of stress under stresses applied to some area of the half-plane. However, in many cases, in contact problems, displacements are also given along with stresses at the boundary. That is, it is considered as a mixed boundary problem. This case can be found mainly in stamping problems.

Mixed boundary conditions in many cases take these four forms.

1. Normal $p(x)$ and resultant $q(x)$ stresses are given at the boundary of the half-plane.
2. At the boundary of the half-plane, $\overline{u_z}$ (x) normal displacement and $q(x)$ shear stresses or $\overline{u_x}$ (x) shear displacements and $p(x)$ normal stresses are given. Such boundary conditions correspond to cases where there are no frictional forces between the interacting surfaces and arise from the geometric profile of the surfaces.

3. At the boundary of the half-plane, $\overline{u_z}(x)$, $\overline{u_x}(x)$ normal and tangential displacements are given, that is, it is assumed that there is no sliding of the two surfaces relative to each other.
   In this case, the friction forces between the two surfaces are sufficiently large, and it is necessary to determine the normal and shear stresses at the boundary.
4. Given the normal stress at the boundary, $q(x) = \pm\mu p(x)$ connection is considered between the normal and test stresses. It has a friction coefficient of μ.

The study of the state of stress under the influence of forces applied along a straight line section of an elastic half-plane is one of the important directions of contact problems. Since the loading area in this problem is small compared to the dimensions of the body, the elastic medium is considered semi-infinite, which significantly simplifies the solution.

As a result of the analysis, it is determined that under the influence of distributed normal and shear loads, a complex field of normal and shear stresses is formed in the half-plane. The value and distribution pattern of stresses depend on the intensity of the load, the nature of the distribution, and the coordinates of the point under consideration. At points close to the loaded area, the stresses have the largest values, and their effect gradually decreases with increasing depth.

The solutions obtained in the polar and Cartesian coordinate systems allow determining the stress and displacement components. This is of great importance in assessing contact pressures, calculating deformations, and analyzing the strength of structures.

Thus, the study of the state of stress under the influence of forces applied along a straight line section of an elastic half-plane serves as a theoretical basis for solving contact problems of the theory of elasticity and is practically applied in many engineering fields, such as mechanical engineering, structural mechanics, transport structures, and geotechnics. These results are of important scientific and practical importance for assessing the mechanical state of complex contact systems and increasing their reliability.

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