MUSBAT BLOK MATRITSA.

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Annotatsiya. Ushbu maqolada Musbat aniqlangan Blok matritsasi haqida tushuncha , Musbat aniqlangan matritsasini qo'llash , Musbat aniqlangan matritsasiga oid bir nechta teoremalar va misollar berildi.

Annotation. Annotation. This paper provides an overview of the Positive Block matrix, the application of the Positive matrix, and several theorems and examples related to the Positive matrix.

Kalit so'zlar: Blok matritsa , argument, kompleks, matritsa, transponerlash, musbat matritsa, funksiya, simetrik

Key words: Block matrix, argument, complex, matrix, transponder, positive matrix, function, symmetric

Quyudagi ko'rinishdagi $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ 2×2 blok matritsaning, musbat matritsalarni o'rganishdagi xossallarini ko'rib chiqamiz.

$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ blok matritsaning A, B, C, D elementlari $n \times n$ matritsalardan iborat. Shunday qilib, blok matritsaning kengaytirilgan (kata) matritsasi M_{2n} yoki $\mathcal{L}(\mathcal{H} \oplus \mathcal{H})$ ning elementidir. Biz davom ettirib A ning bir qancha xossalarini olish mumkinligini ko'ramiz A yozuvlardan biri bo'lgan blok matritsasi. Maxsus muhimligi shundaki, bu musbatligi (algebraik xususiyat) va kontraktlik (metrik xususiyat).

Kerakli tushunchalarni kiritamiz. A matritsaning qutb parchalanish ifodasi uchun $A = U \cdot P$ yozamiz, bunda U unitar, P esa musbat matritsa, 1.3.4-xossaga ko‘ra $P = (A^* \cdot A)^{\frac{1}{2}}$ ekanligi ma‘lum. Bu A ning musbat qiymati yoki absolut qiymati deyiladi va $|A|$ shaklida yoziladi. Biz $A^* = P \cdot U^*$ ga egamiz va

$$|A^*| = (A \cdot A^*)^{\frac{1}{2}} = (U \cdot P^2 \cdot U^*)^{\frac{1}{2}} = U \cdot P \cdot U^*$$

Agar $A \cdot A^* = A^* \cdot A$ bo‘lsa, A normal matritsa deyiladi . Bu shart ekvivalent $U \cdot P = P \cdot U$; va $|A| = |A^*|$ sharti.

A ning yagona qiymat qismlarga ajratish (SVD) uchun $A = U \cdot S \cdot V$ yozamiz. Bu erda U va V birlik, S esa manfiy bo‘lmagan diagonaldir diagonal yozuvlar $s_1(A) \geq \cdots \geq s_n(A)$. Bu A ning yagona qiymatlari (A ning xos qiymatlari). $\|A\|$ belgisi har doim $\mathcal{H}$ Gilbert fazosida chiziqli operaor A ning normasini sifatida tasvirlanadi ; ya‘ni,

$$\|A\| = \sup_{|x|=1} \|Ax\| = \sup_{|x|\leq 1} \|Ax\|$$

Bundan $|A| = s_1(A)$ ekanligini ko‘rish mumkin.

Ushbu normaning muhim xususiyatlari qatoriga quyidagilar kiradi:

$$\|AB\| \leq \|A\|\|B\|,$$

$$\|A\| = \|A^*\|$$

$$\|A\| = \|U \cdot A \cdot V\| \text{ unitar } U, V \text{ u} \tag{1}$$

oxirgi xossa unitar o‘zgaruvchanlik deb ataladi. Bundan

$$\|A^* \cdot A\| = \|A\|^2 \tag{2}$$

M_n da ko‘pgina boshqa normalar mavjud, uchta xususiyat (1) da ko‘rsatilindi. Operator normasini $\|\cdot\|$ qiladigan shart (2) juda muhim.

Agar $\|A\| \leq 1$ bo‘lsa, A qisqaruvchi deymiz.

1-xossa agar operator $\begin{bmatrix} I & A \\ A^* & I \end{bmatrix}$ musbat bo‘lsa u holda A operatori qisqaruvchi bo‘ladi.

Isbot. $\mathcal{H}$ bir o‘lchovli bo‘lganda teorema qanday bo‘ladi?

Shunchaki, agar a kompleks son bo‘lsa, u holda $|a| \leq 1$ agar va faqat agar 2×2 matritsa $\begin{bmatrix} 1 & a \\ \bar{a} & 1 \end{bmatrix}$ musbat bo‘lsa. Birdan ko‘pga o‘tish o‘lchamlari SVD orqali amalga oshiriladi. $A = USV$ bo‘lsin. Keyin

$$\begin{bmatrix} I & A \\ A^* & I \end{bmatrix} = \begin{bmatrix} I & U\cdot S\cdot V \\ V^*\cdot S\cdot U^* & I \end{bmatrix} = \begin{bmatrix} U & O \\ O & V^* \end{bmatrix}\begin{bmatrix} I & S \\ S & I \end{bmatrix}\begin{bmatrix} U^* & O \\ O & V \end{bmatrix}.$$

Bu matritsa $\begin{bmatrix} I & S \\ S & I \end{bmatrix}$ unitar ekvivalent, qaysiki to‘g‘ridan-to‘g‘ri yig‘indiga teng

$$\begin{bmatrix} 1 & s_1 \\ s_1 & 1 \end{bmatrix} \oplus \begin{bmatrix} 1 & s_2 \\ s_2 & 1 \end{bmatrix} \oplus \cdots \oplus \begin{bmatrix} 1 & s_n \\ s_n & 1 \end{bmatrix},$$

Bu yerda $s_1, \dots, s_n$ lar A ning yagona qiymatlari. Bu 2×2 matritsalar $s_1 \leq 1$ (ya‘ni, $\|A\| \leq 1$) bo‘lsa, hammasi musbat bo‘ladi.

1-misol. $\begin{bmatrix} I & A \\ A^* & I \end{bmatrix} = I_2$ matritsa berilgan bo‘lsa, matritsani parchalanishga tekshiring.

$\begin{bmatrix} I & A \\ A^* & I \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ birlik ya’ni musbat matritsa .

$$\begin{bmatrix} I & A \\ A^* & I \end{bmatrix} = \begin{bmatrix} I & U\cdot S\cdot V \\ V^*\cdot S\cdot U^* & I \end{bmatrix} = \begin{bmatrix} U & O \\ O & V^* \end{bmatrix}\begin{bmatrix} I & S \\ S & I \end{bmatrix}\begin{bmatrix} U^* & O \\ O & V \end{bmatrix}$$

$A = U\cdot S\cdot V$ liini inobatga olsak $S = [1]$ $U\, va\, V$ unitary $U = [i]$ va $V = [-i]$ deb olib parchalanishni ko‘rsatamiz

$A = [i][1][-i] = -i^2 = 1$ o‘rinli

$$\begin{bmatrix} I & A \\ A^* & I \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

U holda 2- xossa ko‘ra A pachalanish.

1-teorema. A, B musbat matritsalar bo‘lsin. $\begin{bmatrix} A & X \\ X^* & B \end{bmatrix}$ blok matritsa musbat bo‘lishi uchun ba‘zi bir K parchalanish uchun $X = A^{\frac{1}{2}}\cdot K\cdot B^{\frac{1}{2}}$ bo‘lishi kerak.

Isbot. A, B qat‘iy musbat matritsa bo‘ladi deb faraz qilaylik. U holda bu quyudagi moslikdan foydalanishga imkon beradi

$$\begin{bmatrix} A & X \\ X^* & B \end{bmatrix} \sim \begin{bmatrix} A^{-\frac{1}{2}} & O \\ O & B^{-\frac{1}{2}} \end{bmatrix}\begin{bmatrix} A & X \\ X^* & B \end{bmatrix}\begin{bmatrix} A^{-\frac{1}{2}} & O \\ O & B^{-\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} I & B^{-\frac{1}{2}}\cdot X\cdot A^{-\frac{1}{2}} \\ B^{-\frac{1}{2}}\cdot X^*\cdot A^{-\frac{1}{2}} & I \end{bmatrix}$$

$K = A^{-\frac{1}{2}} \cdot X \cdot B^{-\frac{1}{2}}$ berilgan bo‘lsin. So‘ng 1 xossadan bu blok matritsa musbat bo‘ladi, faqatgina K parchalanish bo‘lsa. Bu teoremani A, B qat‘iy musbat bo‘lgan hol uchun isbotlaydi. Umumiy holat quyidagicha davomiylik argumenti bilan.

1-teoremadan kelib chiqadiki, agar $\begin{bmatrix} A & X \\ X^* & B \end{bmatrix}$ musbat bo‘ladi, X rangi A rangining qism fazosi va X^* rangi B rangining qism fazosi bo‘lsa. A rangi yoki B rangi X ning rangidan ham oshmasligi kerak.

2-teorema. A, B qat‘iy musbat matritsalar berilgan bo‘lsin. Agar $A \geq X \cdot B^{-1} \cdot X^*$ bo‘lsa $\begin{bmatrix} A & X \\ X^* & B \end{bmatrix}$ blok matritsasi musbat bo‘ladi.

Isbot. Bizda quyudagi bor

$$\begin{bmatrix} A & X \\ X^* & B \end{bmatrix} \sim \begin{bmatrix} I & -X \cdot B^{-1} \\ O & I \end{bmatrix} \begin{bmatrix} A & X \\ X^* & B \end{bmatrix} \begin{bmatrix} I & O \\ -B^{-1} \cdot X^* & I \end{bmatrix} = \begin{bmatrix} A - X \cdot B^{-1} \cdot X^* & O \\ O & B \end{bmatrix}$$

Shubhasiz, bu oxirgi matritsa faqat $A \geq XB^{-1}X^*$ bo‘lsa musbat bo‘ladi.

Ikkinchi isbot. Bizda $A \geq XB^{-1}X^*$ mavjud agar va faqat agar

$$I \geq A^{-\frac{1}{2}} \cdot (XB^{-1}X^*)A^{-\frac{1}{2}} = A^{-\frac{1}{2}}XB^{-\frac{1}{2}} \cdot B^{-\frac{1}{2}}X^*A^{-\frac{1}{2}} = \left(A^{-\frac{1}{2}}XB^{-\frac{1}{2}}\right)\left(A^{-\frac{1}{2}}XB^{-\frac{1}{2}}\right)^*$$

Bu $\left\|A^{-\frac{1}{2}}XB^{-\frac{1}{2}}\right\| \leq 1$ yoki $X = A^{\frac{1}{2}}KB^{\frac{1}{2}}$ bu yerda $\|K\| \leq 1$ deyishga teng. Endi 2 teoremadan foydalanamiz.

1-lemma. A matritsasi musbat bo‘ladi faqatgina $\begin{bmatrix} A & A \\ A & A \end{bmatrix}$ musbat bo‘lsa.

Isbot. U holda bu ifodani quyudagi ko‘rinishda yozishimiz mumkin

$$\begin{bmatrix} A & A \\ A & A \end{bmatrix} = \begin{bmatrix} A^{\frac{1}{2}} & O \\ A^{\frac{1}{2}} & O \end{bmatrix} \begin{bmatrix} A^{\frac{1}{2}} & A^{\frac{1}{2}} \\ O & O \end{bmatrix}$$

1-xulosa. A biron matritsa berilgan bo‘lsin. U holda $\begin{bmatrix} |A| & A^* \\ A & |A^*| \end{bmatrix}$ matritsa musbat bo‘ladi.

Isbot. $A = UP$ qutb parchalanishi foydalanin yozamiz

$$\begin{bmatrix} |A| & A^* \\ A & |A^*| \end{bmatrix} = \begin{bmatrix} P & P \cdot U^* \\ U \cdot P & U \cdot P \cdot U^* \end{bmatrix} = \begin{bmatrix} I & O \\ O & U \end{bmatrix} \begin{bmatrix} P & P \\ P & P \end{bmatrix} \begin{bmatrix} I & O \\ O & U^* \end{bmatrix}.$$

va keyin lemmadan foydalanamiz.

2-misol. $A = \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$ matritsa berilgan bo‘lsin, u holda $\begin{bmatrix} |A| & A^* \\ A & |A^*| \end{bmatrix}$ musbatlikda tekshiringa

$$\begin{bmatrix} |A| & A^* \\ A & |A^*| \end{bmatrix} = \begin{bmatrix} I & O \\ O & U \end{bmatrix} \begin{bmatrix} P & P \\ P & P \end{bmatrix} \begin{bmatrix} I & O \\ O & U^* \end{bmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & i & 1 & i \\ i & 1 & i & 1 \\ 1 & i & 1 & i \\ i & 1 & i & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & i & 1 & i \\ i & 1 & i & 1 \\ 1 & i & 1 & i \\ i & 1 & i & 1 \end{pmatrix} = \begin{bmatrix} P & P \\ P & P \end{bmatrix}$$

1 lemmaga ko‘ra $\begin{bmatrix} P & P \\ P & P \end{bmatrix}$ musbat aniqlanganligidan, demak $\begin{bmatrix} |A| & A^* \\ A & |A^*| \end{bmatrix}$ musbat anilangan.

2-xulosa. Agar A matritsa normal bo‘lsa, u holda $\begin{bmatrix} |A| & A^* \\ A & |A^*| \end{bmatrix}$ musbat bo‘ladi.

3-misol. Agar $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$ berilgan bo‘lsa, u holda $\begin{bmatrix} |A| & A^* \\ A & |A^*| \end{bmatrix}$ matritsani musbatlikga tekshiring

Dastavval A matritsani normallikda teshirsak ya’ni $AA^T = A^T A$ o‘rinli ekanini ko‘rsatamiz

$$A^T = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad AA^T = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$ bundan A matritsa normal

1 –xulosadan foydalansak

$$\begin{bmatrix} |A| & A^* \\ A & |A^*| \end{bmatrix} = \begin{bmatrix} I & O \\ O & U \end{bmatrix} \begin{bmatrix} P & P \\ P & P \end{bmatrix} \begin{bmatrix} I & O \\ O & U^* \end{bmatrix} \quad A = PU = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\begin{bmatrix} |A| & A^* \\ A & |A^*| \end{bmatrix} = \begin{bmatrix} I & O \\ O & U \end{bmatrix} \begin{bmatrix} P & P \\ P & P \end{bmatrix} \begin{bmatrix} I & O \\ O & U^* \end{bmatrix} = \begin{bmatrix} I & O \\ O & U \end{bmatrix} \begin{bmatrix} I & O \\ O & U^* \end{bmatrix} = \begin{bmatrix} I & O \\ O & I \end{bmatrix}$$

I_4 birlik matritsaga keladi bu matritsa musbat aniqlanganidan Blok matritsa musbat aniqlanganligi kelib chiqadi.

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