STRESS STATE OF A CIRCULAR PLATE WITH RADIALLY VARYING THICKNESS UNDER ANTISYMMETRIC LOADING.

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Abstract. In this article, the stress-strain state of a circular plate whose thickness varies along the radius under antisymmetric loading is numerically investigated on the basis of the finite difference method. The governing differential equation of plate bending is reduced to a discrete form, and finite-difference analogues of the boundary conditions are obtained. Using graphs, the distribution laws of deflections, bending moments, and transverse forces are determined for plates made of aluminum and steel materials. The obtained results confirm that the radial variation of thickness and the elastic properties of the material have a significant effect on the mechanical state of the structure.

Keywords: circular plate, finite difference method, deflection, moment, transverse force, numerical modeling, variable thickness.

Introduction. Plate and shell structures are widely used in modern mechanical engineering, aviation, energy, and construction. In order to increase the strength and economic efficiency of these structures, the use of elements with variable thickness is one of the important current issues. Circular plates with thickness varying along the radius can reduce weight while ensuring the required stiffness.

Since obtaining analytical solutions of the governing differential equations for such structures is complicated, it is appropriate to use numerical methods. In this work, the deformation of a circular plate with radially varying thickness under antisymmetric loading was studied using the finite difference method.

Problem statement. In an axisymmetric plate with thickness varying in the radial direction, the stiffness does not depend on the polar coordinate θ, that is,

The differential equation of plate bending is expressed in the following form:

$$D\Delta\Delta w+2\frac{\partial D}{\partial r}\frac{\partial}{\partial r}\Delta w+\left(\frac{\partial^2 D}{\partial r^2}+\frac{1}{r}\frac{\partial D}{\partial r}\right)\Delta w-(1-\nu)\times$$
$$\times\left[\frac{\partial^2 D}{\partial r^2}\left(\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \phi^2}\right)+\frac{1}{r}\frac{\partial D}{\partial r}\frac{\partial^2 D}{\partial r^2}\right]+Cw=P(r,\phi) \qquad (1)$$

The expressions for the moments and the transverse forces expressed through them are represented in terms of the deflection function in the following form.

$$M_1=-D\left[\frac{\partial^2 w}{\partial r^2}+\nu\left(\frac{1}{r^2}\frac{\partial^2 w}{\partial \phi^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)\right];\quad M_2=-D\left(\nu\frac{\partial^2 w}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \phi^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right);$$
$$M_{12}=M_{21l}=-D(1-\nu)\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial w}{\partial \phi}\right),\ Q_1=-D\frac{\partial(\Delta w)}{\partial r}+\frac{\partial D}{\partial r}\frac{M_1}{D};Q_2=-D\frac{1}{r}\frac{\partial(\Delta w)}{\partial \phi}+\frac{\partial D}{\partial r}\frac{M_{12}}{D}. \qquad (2)$$

The expression of the reduced transverse forces for a plate of variable thickness takes the following form:

$$V_1=\pm\left(Q_1+\frac{1}{r}\frac{\partial M_{12}}{\partial \phi}\right);\quad V_2=\pm\left(Q_2+\frac{\partial M_{21}}{\partial r}\right) \qquad (3)$$

The upward signs in the equations correspond to the side of the plate located closer to the coordinate origin, while the downward signs correspond to the side located farther from it.

Solution of the problem. We reduce the differential equation of plate bending (1) to the following system of algebraic equations using the finite difference method among numerical methods:

$$w_m F_m+w_e F_e+w_k F_k(w_{k+1}+w_{k-1})F_{k+1}+w_h F_h+w_i F_i+$$
$$+(w_{e+1}+w_{e-1})F_{e+1}+(w_{i+1}+w_{i-1})F_{i-1}+(w_{k+2}+w_{k-2})F_{k+2}=\frac{P_k}{D_k}\Delta r^4 \qquad (4)$$

where

$$F_m = 1 + A_k; F_e = -4 - 2A_k + B_k + \frac{1}{\beta_k^2} - 2H_k;$$

$$F_k = 2\left(3 - B_k + \frac{4}{\beta_k^2} - E_k + \frac{3}{\beta_k^4}\right) + \frac{C\Delta r^4}{D_k};\ F_i = -4 + 2A_k + B_k - C_k - \frac{4}{\beta_k^2} + 2H_k;$$

$$F_{e+1} = \frac{2}{\beta_k^2} + H_k = F_{e-1};\ F_{i+1} = \frac{2}{\beta_k^2} - H_k = F_{i-1};\ F_{k+2} = \frac{1}{\beta_k^4} = F_{k-2}$$

$$F_{k+1} = -\frac{4}{\beta_k^4}\left(\beta_k^2 + 1\right) + E_k = F_{k-1};\quad F_h = 1 - A_k$$

$$A_k = \frac{1}{n_k} + \Delta r\frac{D_k^{/}}{D_k}; B_k = -\frac{1}{n_k^2} + \frac{2+\nu}{n_k}\Delta r\frac{D_k^{/}}{D_k} + \Delta r^2\frac{D_k^{//}}{D_k};$$

$$C_k = \frac{1}{2n_k}\left(\frac{1}{n_k^2} - \frac{\Delta r}{n_k}\frac{D_k^{/}}{D_k} + \nu\nabla r^2\frac{D_k^{//}}{D_k}\right);\ H_k = \frac{1}{\beta_k^2}\left(-\frac{1}{h_k} + \Delta r\frac{D_k^{/}}{D_k}\right);$$

$$E_k = \frac{1}{\beta_k^2}\left(\frac{4}{h_k^2} - \frac{3\Delta r}{n_k}\frac{D_k^{/}}{D_k} + \nu\Delta r^2\frac{D_k^{//}}{D_k}\right).$$

From the numerical solution of the algebraic system of equations (4) obtained using the Maple program, we construct the analytical expression of the deflection function W(r):

$W(r)=6.05065805\times10^{-13}r^5-3.83931541\times10^{-12}r^4+8.90056911\times10^{-12}r^3-8.76888691\times10^{-12}r^2+3.23364225\times10^{-12}r$ (5)

Let the thickness variation be as follows:

$h(r)=0.006666667r^3-0.001r^2-0.150166667r+0.3485$ (6)

From (6) and according to $D = \frac{Eh^3}{12(1-\nu^2)}$, the stiffness function is:

$D(r)=\frac{Eh^3}{12(1-\nu^2)}(0.006666667r^3-0.001r^2-0.150166667r+0.3485)$ (7)

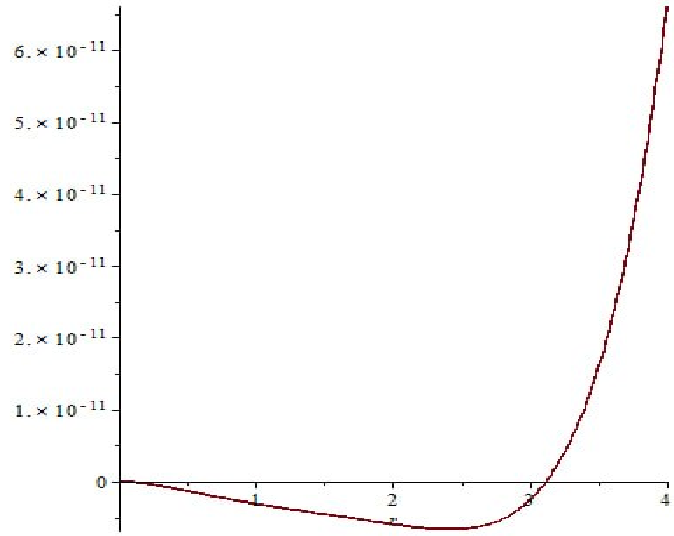

Figure 1. Graph of the function W(r) (aluminum)

Figure 1 shows the variation of the deflection function W(r) along the radius for a circular plate of variable thickness made of aluminum. It can be seen from the graph that, as one moves from the center of the plate toward the edge, the value of deflection changes according to a nonlinear law. This situation is explained by the change in stiffness as a result of the variation of the plate thickness along the radius. Since the stiffness is higher in the central part, the deflection has small values, and as the outer regions are approached, the rate of change of the deflection increases.

The graphs of the moments and the transverse forces expressed through them were determined using the found deflection function (5) in the following form:

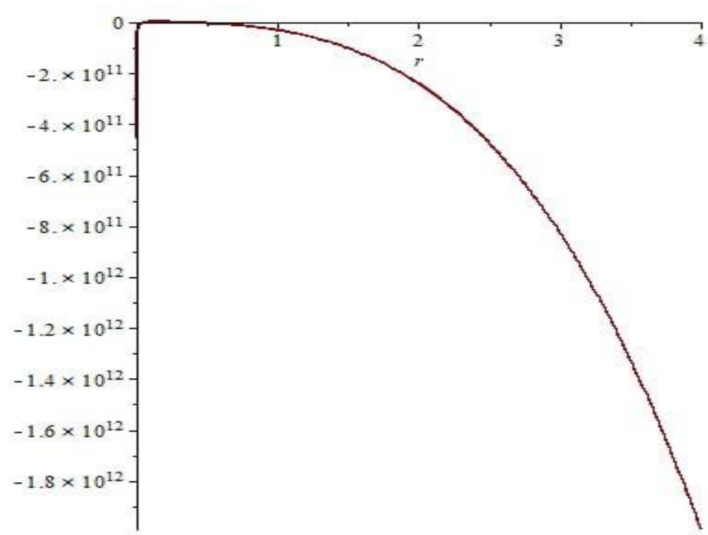

Figure 2. Graph of the moment M_1 (aluminum)

Figure 2 presents the distribution of the bending moment M_1 in the radial direction. The analysis of the graph shows that the moment values are not distributed uniformly along the radius. As a result of the decrease in the plate thickness, the intensity of the moments changes and reaches maximum values in certain regions. This indicates that stress concentration may occur in the structure.

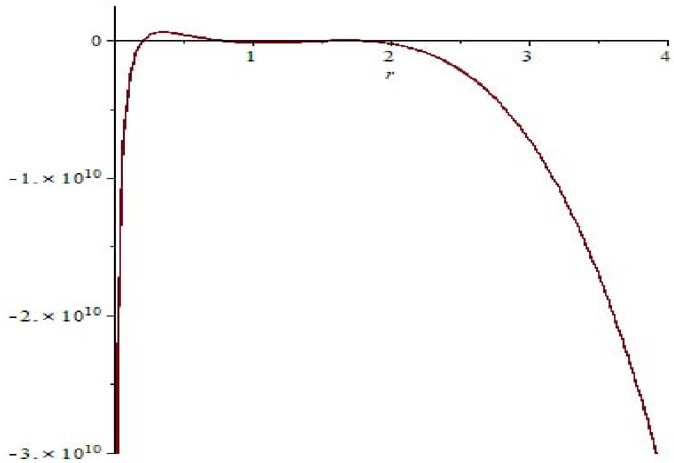

Figure 3. Graph of the moment M_2 (aluminum)

Figure 3 shows the graph of the variation of the moment M_2 in the circumferential direction. The results show that although the distribution character of the moment M_2 is close to that of the moment M_1, their numerical values and extremal points differ from each other. This difference is related to the geometric characteristics of the plate and to the radial variation of the stiffness function.

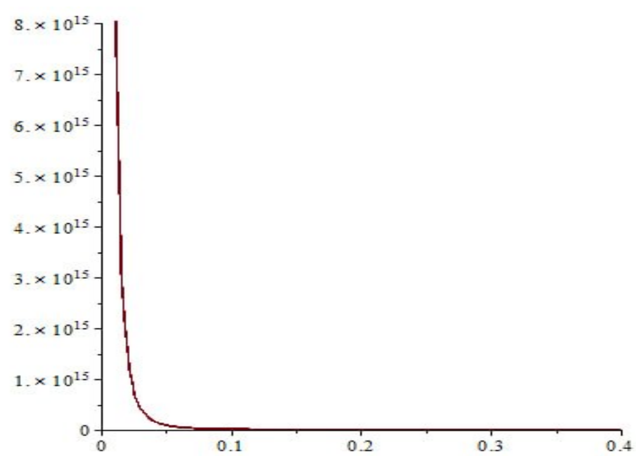

Figure 4. Graph of the transverse force V_1 (aluminum)

Figure 4 presents the distribution of the transverse force V_1 along the radius. It can be seen from the graph that the transverse forces change sharply in certain parts of the plate. In particular, in the zones where the thickness changes rapidly, the force gradients increase, and the probability of additional local stresses occurring in these regions becomes high.

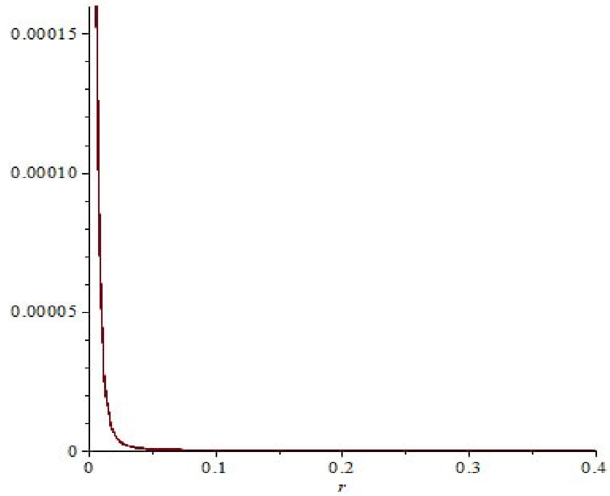

Figure 5. Graph of the transverse force V_2 (for aluminum and steel)

Figure 5 gives a comparative graph of the transverse forces V_2 for plates made of aluminum and steel materials. The comparison results show that, because the modulus of elasticity of the steel plate is higher, the overall stiffness of the structure increases and the deformations decrease. In the aluminum plate, due to the relatively smaller modulus of elasticity, the deflections are larger, and noticeable differences are also observed in the distribution of transverse forces. These results confirm that the mechanical properties of the material directly affect the stress-strain state of the plate.

Conclusion. Based on the calculation results, the distribution laws of the deflection function, radial and circumferential moments, and transverse forces were determined. The research results showed that the variation of the plate thickness along the radius has a significant effect on the stiffness of the structure; as a result, the distribution of deflections and internal force factors acquires a nonlinear character. In addition, the results obtained for aluminum and steel materials confirmed that, as the modulus of elasticity increases, the deformations decrease and the stiffness of the structure increases.

The obtained results have practical significance in selecting optimal geometric parameters, reducing material consumption, and ensuring strength and stiffness requirements in the design of plate structures with variable thickness. In the future, this approach can be developed for complex loadings, plates on an elastic foundation, and shell-like structures.

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