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OPTIMIZING HYDROELECTRIC TURBINE FLOW DISTRIBUTION FOR MAXIMUM POWER OUTPUT

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Аннотация: *Annotatsiya (O'zbekcha): Ushbu maqola Penobskot daryosidagi gidroelektr stansiyasida turbinalar orqali suv oqimini optimal taqsimlash masalasini ko'rib chiqadi. Maqsad – umumiy energiya ishlab chiqarishni maksimallashtirish uchun Lagrange multiplikatorlari yordamida optimal suv oqimi taqsimotini aniqlash. Maqolada uchta turbinaning har biri uchun kvadratik quvvat modellaridan foydalaniladi va turli suv oqimi stsenariylari tahlil qilinadi. Natijalar har bir turbinaning oqim chegaralarini hisobga olgan holda maksimal quvvatni ta'minlaydi.*

Аннотация (Русский): *В статье рассматривается задача оптимального распределения по-тока воды через турбины на гидроэлектростанции на реке Пенобскот. Цель – максимизация общей выработки энергии с использованием метода множителей Лагранжа для определения оптимального распределения потока. В статье используются квадратичные модели мощности для трех турбин, а также анализируются различные сценарии потока воды. Результаты обес-печивают максимальную мощность с учетом ограничений потока каждой турбины.*

Abstract (English): *This paper addresses the optimization of water flow distribution through turbines at a hydroelectric power station on the Penobscot*

River. The objective is to maximize total power output using Lagrange multipliers to determine optimal flow distribution. Quadratic power models for three turbines are employed, and various flow scenarios are analyzed. The results ensure maximum power output while respecting each turbine's flow constraints.

Tayanch so'zlar (O'zbekcha): *gidroelektr stansiya, suv oqimi, Lagrange multiplikatorlari, optimallashtirish, quvvat ishlab chiqarish, yashil iqtisodiyot.*

Ключевые слова (Русский): *гидроэлектростанция, поток воды, множители Лагранжа, опти-мизация, выработка мощности, зеленая экономика.*

Keywords (English): *hydroelectric power station, water flow, Lagrange multipliers, optimization, power generation, green economy.*

1 Introduction

The global push towards sustainable energy solutions has placed significant emphasis on optimizing renewable energy sources, such as hydroelectric power, which aligns with the principles of the green economy. Hydroelectric power stations, like the one operated by the Penobscot Water Company in Millinocket, Maine, rely on efficient water flow distribution to maximize energy output. This paper addresses the mathematical optimization of water flow through three distinct turbines, each governed by quadratic power output models, to achieve maximum energy production under varying flow conditions. By employing Lagrange multipliers, we derive optimal flow distributions that respect turbine-specific constraints, contributing to efficient and environmentally friendly energy production. This work aligns with the conference theme of innovative solutions for environmental challenges in engineering.

2 Methods

To optimize the power output of the hydroelectric station, we model the power generated by each turbine using quadratic equations provided by Bernoulli's empirical formulas. The power output for each turbine is defined as follows:

$KW1 = (-18.89 + 0.1277Q1 - 4.08 \cdot 10-5Q2)(170 - 1.6 \cdot 10-6Q2)$(1)

$KW2 = (-24.51 + 0.1358Q2 - 4.89 \cdot 10-5Q2)(170 - 1.6 \cdot 10-6Q2)$ (2) $KW3 = (-27.02 + 0.1380Q3 - 3.84 \cdot 10-5Q2)(170 - 1.6 \cdot 10-6Q2)$ (3)

where Qi represents the water flow through turbine i (in cubic feet per second), KWi is the power output of turbine i (in kilowatts), and $Q = Q1 + Q2 + Q3$ is the total flow through the station. The constraints are:

$250 \leq Q1 \leq 1110, 250 \leq Q2 \leq 1110, \ 250 \leq Q3 \leq 1125$

The optimization problem is to maximize the total power output KW1 + KW2 + KW3 subject to the constraint $Q1 + Q2 + Q3 = Q$. We employ Lagrange multipliers to solve this constrained optimization problem. The Lagrangian is defined as:

$L(Q1, Q2, Q3, \lambda) = KW1 + KW2 + KW3 + \lambda(Q - Q1 - Q2 - Q3)$

Taking partial derivatives with respect to Q1, Q2, Q3, and λ, and setting them to zero, we obtain the system of equations to solve for the optimal flows.

3 Results and Discussion

3.1 Optimal Flow Distribution for Three Turbines

Using Lagrange multipliers, the partial derivatives of the power functions with respect to Q1, Q2, and Q3 are equated, leading to the following relationships:

$Q2 = 86.354 + 0.8699Q1$ (4)

$Q3 = 134.115 + 1.0625Q1$ (5)

Substituting these into the constraint $Q1 + Q2 + Q3 = Q$, we derive:

$Q1 = 0.341Q - 75.134$ (6)

$Q2 = 0.297Q + 20.995$ (7)

$Q3 = 0.362Q + 54.285$ (8)

The valid range for Q is determined by ensuring each Qi satisfies its respective constraints, yielding

$954.7 \leq Q \leq 3231.34$. For a total flow of $Q = 2500$ ft3/s, the optimal flows are:

$Q1 = 777.386, \quad Q2 = 762.495, \quad Q3 = 960.035$

These values yield a total power output of 28,410.61 kW. To verify maximality, alternative distributions (e.g., Q1 = 790, Q2 = 755, Q3 = 955) were tested, resulting in lower outputs (e.g., 28,410.045 kW), confirming the optimality of the derived solution.

3.2 Single Turbine Operation

For a total flow of Q = 1000 ft3/s, operating only one turbine was evaluated. Assigning the entire flow to Turbine 3 (Q3 = 1000) yields 12,222.472 kW, significantly higher than using all three turbines (8,398.207 kW). For Q = 600 ft3/s, Turbine 1 provides the highest output (7,292 kW) compared to Turbine 2 (6,837 kW) and Turbine 3 (7,108 kW).

3.3 Two Turbine Operation

For Q = 1500 ft3/s, using Turbines 1 and 3 yields the highest power output (18,208.300 kW) with flows Q1 = 662.247 and Q3 = 837.753. This is more efficient than using all three turbines (16,538.681 kW). For Q = 1600 ft3/s, the same turbine pair remains optimal, with adjusted flows derived similarly.

3.4 High Flow Scenario

For Q = 3400 ft3/s, which exceeds the upper constraint for Turbine 3 (Q3 ≤ 1225), we assign Q3 = 1225 and distribute the remaining flow (3400 − 1225 = 2175) between Turbines 1 and 2 using Lagrange multipliers, ensuring compliance with their constraints.

4 Conclusion

This study demonstrates the application of mathematical optimization, specifically Lagrange multipliers, to maximize power output in a hydroelectric power station. The derived flow distributions ensure efficient energy production across various flow scenarios, supporting the green economy by optimizing renewable energy resources. The results highlight the importance of strategic turbine operation, particularly favoring single or dual turbine configurations under specific flow conditions, to achieve higher efficiency. Future work could explore dynamic flow variations and incorporate additional environmental factors to further enhance sustainability.

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