



MURAKKAB FUNKSIYALARNI HOSILASINI HISOBLASH

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Annotatsiya: Hosila mavzusi o’rta maktab, akademik litseylarda o’tilib o’quvchilarga ma’lum bir tushunchalar beriladi. Lekin hozirgi kunda davlat test markazlaridan shu boblar uchun qo’yilgan talablar anchagina murakkab.

Ushbu mavzuda esa murakkab funksiyalarning hosilalari natural logarifm yordamida oson va qulay usulda ko’rsatilgan.

Kalit so’zlar: Natural logarifm, hosila, yechim, masala.

Hosila yordamida matematikada juda ko’p masalalarni hal qilish mumkin. Ushbu mavzuda murakkab funksiyalarni natural logarifm yordamida hosilasini olish oson va qulay holda ko’rsatamiz $y = \ln(f(x))$ funksiyani o’zini aniqlanish sohasida hosilasi:

$$(\ln f(x))' = \frac{f'(x)}{f(x)} \text{ ga teng.}$$

Misol-1. $y = x^{\sin 2x}$ funksiyaning hosilasini toping?

Yechish: Tenglikni har ikkala tomonini natural logarifmlaymiz:

$$\ln y = \ln x^{\sin 2x}$$

$$\ln y = \sin 2x \ln x \quad (1)$$

(1) tenglikni har ikkala tomonidan hosila olamiz:

$$(\ln y)' = (\sin 2x \cdot \ln x)'$$

$$\frac{y'}{y} = 2 \cos 2x \cdot \ln x + \frac{\sin 2x}{x}$$

$$y' = y \cdot \left(2 \cos 2x \cdot \ln x + \frac{\sin 2x}{x} \right) \text{ bundan}$$

$$y' = x^{\sin 2x} \cdot \left(2 \cos 2x \cdot \ln x + \frac{\sin 2x}{x} \right) \text{ ga teng.}$$



Javob: $(x^{\sin 2x})' = x^{\sin 2x} \cdot (2 \cos 2x \cdot$

$$\ln x + \frac{\sin 2x}{x})$$

Misol-2. $y = (\operatorname{tg} 3x)^{\sin 6x}$ funksiyani hosilasini toping?

Yechish: Tenglikni har ikkala tomonini natural logarifmlaymiz:

$$\ln y = \ln(\operatorname{tg} 3x)^{\sin 6x}$$

$$\ln y = \sin 6x \cdot \ln(\operatorname{tg} 3x)$$

funksiyani hosilasini olamiz:

$$(\ln y)' = (\sin 6x \cdot \ln(\operatorname{tg} 3x))'$$

$$\frac{y'}{y} = 6 \cos 6x \cdot \ln(\operatorname{tg} 3x) + \frac{3}{\cos^2 3x \cdot \operatorname{tg} 3x} \cdot \sin 6x$$

$$\frac{y'}{y} = 6 \cos 6x \cdot \ln(\operatorname{tg} 3x) + \frac{6}{\sin 6x} \cdot \sin 6x = 6(\cos 6x \cdot \operatorname{tg} 3x + 1)$$

$$y' = 6(\operatorname{tg} 3x)^{\sin 6x} (\cos 6x \cdot \operatorname{tg} 3x + 1)$$

Javob:

$$6(\operatorname{tg} 3x)^{\sin 6x} (\cos 6x \cdot \operatorname{tg} 3x + 1)$$

Misol-3. $y = (x + 1)^{\sqrt{x}}$ funksiya berilgan bo'lsa $y'(1)$ ni hisoblang.

Yechish: Yuqoridagi shartlarga ko'ra

$$\ln y = \ln(x + 1)^{\sqrt{x}}$$

$$(\ln y)' = (\ln(x + 1)^{\sqrt{x}})' \text{ bundan}$$

$$\frac{y'}{y} = (\sqrt{x} \cdot \ln(x + 1))' = \frac{1}{2\sqrt{x}} \cdot \ln(x + 1) + \sqrt{x} \cdot \frac{1}{x + 1}$$

$$y' = (x + 1)^{\sqrt{x}} \cdot \left(\frac{\ln(x + 1)}{2\sqrt{x}} + \frac{\sqrt{x}}{x + 1} \right) \text{ endi } y'(1) \text{ ni hisoblaymiz:}$$

$$y'(1) = 2 \left(\frac{\ln 2}{2} + \frac{1}{2} \right) = \ln 2 + 1 = \ln 2e$$

Javob: $\ln 2e$

Misol-4. $y = \sqrt[5]{(x + 2)^2(x^2 - 1)^3\sqrt{x - 4}}$ funksiyaning hosilasini hisoblang.

Yechish: Tenglikni har ikkala tomonini natural logarifmlaymiz:



$$\ln y = \ln \left(\sqrt[5]{(x+2)^2} (x^2-1)^3 \sqrt{x-4} \right) \quad \text{tenglikni} \quad \text{o'ng} \quad \text{tomonini}$$

logarifmning xossalariga ko'ra:

$$\ln y = \ln \sqrt[5]{(x+2)^2} + \ln(x^2-1)^3 + \ln \sqrt{x-4} \quad \text{keltirib hosila olamiz:}$$

$$\frac{y'}{y} = \frac{2}{5} \cdot \frac{1}{x+2} + \frac{6x}{x^2-1} + \frac{1}{2(x-4)}$$

$$y' = y \left(\frac{2}{5(x+2)} + \frac{6x}{x^2-1} + \frac{1}{2(x-4)} \right) \quad \text{bundan}$$

$$y' = \sqrt[5]{(x+2)^2} (x^2-1)^3 \sqrt{x-4} \cdot \left(\frac{2}{5(x+2)} + \frac{6x}{x^2-1} + \frac{1}{2(x-4)} \right) \quad \text{ga} \quad \text{teng}$$

bo'ladi.

Misol-5. $y = \frac{e^{x \arcsin x}}{x^2-1}$ funksiyani hosilasini hisoblang.

Yechish: Yuqoridagi kabi $\ln y = \ln \frac{e^{x \arcsin x}}{x^2-1}$

$$\ln y = \ln e^x + \ln(\arcsin x) - \ln(x^2-1)$$

tenglikni har ikkala tomonidan hosila olamiz:

$$\frac{y'}{y} = 1 + \frac{1}{\arcsin x \sqrt{1-x^2}} - \frac{2x}{x^2-1}$$

$$y' = y \left(1 + \frac{1}{\arcsin x \sqrt{1-x^2}} - \frac{2x}{x^2-1} \right)$$

$$y' = \frac{e^{x \arcsin x}}{x^2-1} \left(1 + \frac{1}{\arcsin x \sqrt{1-x^2}} - \frac{2x}{x^2-1} \right)$$

Javob:

$$y' = \frac{e^{x \arcsin x}}{x^2-1} \left(1 + \right.$$

$$\left. \frac{1}{\arcsin x \sqrt{1-x^2}} - \frac{2x}{x^2-1} \right)$$

Mavzuni mustahkamlash uchun quyidagi funksiyalarni hosilasini hisoblab ko'ring:

$$1. \quad y = (\sin x)^{\sqrt{x^2+2x+5}}$$

$$2. \quad y = (\arcsin(2x+1))^{\cos 2x}$$

$$3. \quad y = \sqrt[3]{(2x+1)^2} (3x+1)^7 (2x-1)^3$$

$$4. \quad y = (\sqrt{1-x^3})^{x^2}$$

$$5. \quad y = \sqrt[5]{\frac{(1-x^2)\cos x}{(x^2+1)^3}} \quad \text{bo'lsa } y'(0) \text{ ni hisoblang.}$$



6. $y = \frac{(2x-1)^3 \sqrt[4]{4x^2-1}}{(2x+1)^3}$ funksiya hosilasining nollarini toping.

Xulosa: Matematika fani o'sib kelayotgan yosh avlodni kamol toptirishda o'quv fani sifatida keng imkoniyatlarga ega. U o'quvchi tafakkurini rivojlantirib, ularni aqlini pishlaydi, uni tartibga soladi, o'quvchilarga maqsadga yo'naltirganlik, mantiqiy fikirlash topqirlik xislatlarini shakillantirib boradi.

Shunday ekan yuqorida keltirilgan ma'lumotlarni matematika mutaxassisligida ta'lim olayotgan talabalar va o'quvchilarni murakkab funksiyalarni hosilalarini hisoblashda qo'yiladigan talablarni oson o'rganishda yordam beradi.

FOYDALANILGAN ADABIYOTLAR

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