TESKARI SIMMETIRIK YUKLANGAN QALINLIGI RADIAL RAVISHDA O'ZGARUVCHI PLASTINKANI CHEKLI AYIRMALAR USULI YORDAMIDA HISOBLASH

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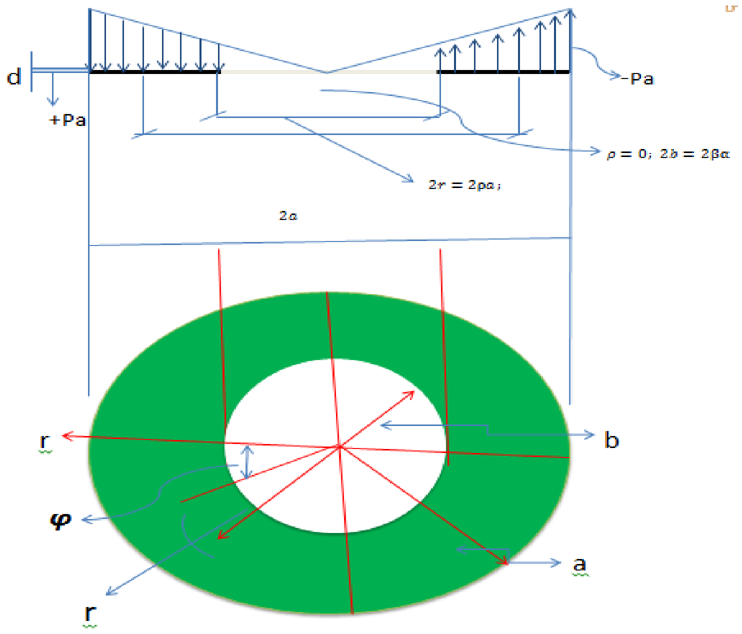
Annotatsiya: Mazkur ishda radius bo'yicha qalinligi o'zgaruvchi doiraviy plastinkaning teskari simmetrik yuklanish ostidagi egilish holati chekli ayirmalar usuli yordamida tahlil qilindi. Masala differensial tenglama ko'rinishida ifodalanib, analitik yechim olish murakkab bo'lganligi sababli sonli usul — chekli ayirmalar usuli qo'llanildi. Natijada differensial tenglama diskret ko'rinishga keltirilib, chegaraviy shartlar asosida yechimlar aniqlandi.

Kalit so'zlar: doiraviy plastinka, o'zgaruvchan qalinlik, simmetrik yuklanish, chekli ayirmalar usuli, chegaraviy shartlar

Kirish. Muhandislik qurilmalarida ko'p ishlatiladigan plastinkalarni qalinligi o'zgarishidan bog'liq kuchlangan-deformatsiyalangan holatlarini zamonaviy hisob usullariga tayanib tadqiq etish va olingan yechimlarni aniqligini baholash qurilish mexanikasining dolzarb masalalari hisoblanadi. Plastinkalarni tashqi ta'sirlar natijasida kuchlangan-deformatsiyalangan holatlarni o'rganish, tekshirish ishlarni tahlil qilish plastinka qalinligiga o'zgarishidan bog'liq deformatsiyalanishini atroflicha bosqichma- bosqich o'rganib chiqish imkonyati mavjud.

Masalaning qo'yilishi. Ichki tomondan qistirib mahkamlangan teskari simmetrik yuklangan qalinligi o'zgaruvchi halqali doiraviy plastinka egilishlari uchun o'rinli tenglamaning sonli usullardan chekli ayirmalar usuli yordamida yechish va olingan yechimlar asosida ko'ndalang kuchlar, eguvchi va burovchi momentlarni aniqlash hamda grafiklari asosida kuchlangan-deformatsiyalangan holatini tahlil qilish masalasi qo'yildi.

Plastinkasi qo'yilgan yuklanish quyidagi formada bo'lsin.



Qalinligi faqat radial yo’nalishda o’zgaruvchi o’qqa nisbatan simmetrik plastinka bikirligi φ koordinata boyicha o’zgarmaydi. Plastinka egilish differensial tenglamasi esa quyidagi ko’rinishda bo’ladi :

$$D\Delta\Delta w + 2\frac{\partial D}{\partial r}\frac{\partial}{\partial r}\Delta w + \left(\frac{\partial^2 D}{\partial r^2} + \frac{1}{r}\frac{\partial D}{\partial r}\right)\Delta w - (1-\mu) \left[\frac{\partial^2 D}{\partial r^2}\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \varphi^2}\right) + \frac{1}{r}\frac{\partial D}{\partial r}\frac{\partial^2 D}{\partial r^2}\right] + Cw = P(r,\varphi) \qquad (1)$$

Qistirib mahkamlanganlik sharti.Agar $r = const$ tomon qistirib mahkamlangan bo’lsa bu tomon uchun w ko’chish va uning hosilasi nolga teng bo’ladi.

$$w = 0; \quad \frac{\partial w}{\partial r} = 0 \qquad (2)$$

Qistirib mahkamlangan plastinkaning $\varphi = const$ tomoni uchun chegaraviy shart esa quyidagicha ifodalanadi.

$$w = 0; \quad \frac{1}{r}\frac{\partial w}{\partial \varphi} = 0 \qquad (3)$$

Qistirib mahkamlangan kontur yoyida burovchi moment nolga teng. Ko’ndalang kuch esa haqiqiy ko’ndalang kuch bilan bir xil bo’ladi.

Masalaning yechilishi. Doiraviy plastinkani rasmdagi kabi radius bo’yicha chiziqli o’zgaruvchi teskari simmetrik qo’yilgan bo’lsin o’lchamsiz radius kiritamiz.

$$\rho = r/a; \quad \beta = b/a; \quad P(r,\varphi) = P\rho\cos\varphi \qquad (4)$$

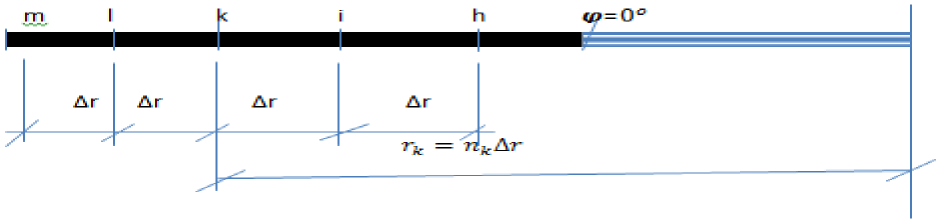
Qalinligi radial yo’nalishda o’zgaruvchi doiraviy plastinkani (4) formada yuklanish tasirida egilishlari uchun o’rinli qalinligi o’zgaruvchi doiraviy plastinka egilish tenglamasi (1) yechimini quyidagi formada olish mumkin.

$$w = W\cos\phi \qquad (5)$$

Bu yerda W faqat r radyusdan bog’liq funksiya. Olingan yechim chegaraviy shartlarni qanoatlantirishi kerak. (5) ni (1) ga qo’yamiz va soddalashtirishlardan keyin quyidagiga ega bo'lamiz.

$$D\left(\frac{d^4W}{dr^4}+\frac{2}{r}\frac{d^3W}{dr^3}-\frac{3}{r^2}\frac{d^2W}{dr^2}+\frac{3}{r^3}\frac{dW}{dr}-\frac{3}{r^4}W\right)+$$
$$+\frac{dD}{dr}\left(2\frac{d^3W}{dr}+\frac{2+\mu}{r}\frac{d^2W}{dr^2}-\frac{3}{r^2}\frac{dW}{dr}+\frac{3}{r^3}W\right)+\frac{d^2D}{dr^2}\left(\frac{d^2W}{dr^2}+\frac{\mu}{r}\frac{dW}{dr}-\frac{\mu}{r^2}W\right)= \qquad (6)$$
$$=P_\alpha\frac{r}{a}-CW$$

Hosil qilingan diferensial tenglama faqat bitta o'zgaruvchidan bog'liq, φ ga bog'liq hadlar yo'qolib ketadi. Bu tenglama yechimini topish murakab shuning uchun tenglamani yechishda taqribiy metodlardan chekli ayirmalar metodidan foydalanamiz. Bu chekli ayirmali metodlarni quyidagicha kiritamiz.



$$\left.\begin{aligned}
&\left(\frac{dW}{dr}\right)_k=\frac{W_e-W_i}{2\Delta r},\quad \left(\frac{d^2W}{dr^2}\right)_k=\frac{W_e-2W_k+W_i}{\Delta r^2}\\
&\left(\frac{d^3W}{dr^3}\right)_k=\frac{W_{me}-2(W_e-W)-W_h}{2\Delta r^3}\\
&\left(\frac{d^4W}{dr^4}\right)_k=\frac{W_m-4W_e+6W_k-4W_i+W_h}{\Delta r^4}
\end{aligned}\right\} \qquad (7)$$

(7) hisobga olgan holda (6) ni quyidagi shaklga keltiramiz:

$$W_mF_m^*+W_eF_e^*+W_kF_k^*+W_iF_i^*+W_hF_h^*=\frac{P_a n_k}{aD_k}\Delta r^5 \qquad (8)$$

Bu tenglamada

$$\left.\begin{aligned}
&F_m^*=1+A_k^*\\
&F_e^*=-4-2A_k^*+B_k^*+C_k^*\\
&F_k^*=6-2B_k^*+E_k^*+C\frac{\Delta r^4}{D_k}\\
&F_i^*=-4+2A_k^*+B_k^*-C_k^*\\
&F_h^*=1-A_k^*
\end{aligned}\right\} \qquad (9)$$

Bu yerda

$$A_k^*=\frac{1}{n_k}+\Delta r\frac{D_k^/}{D_k}$$
$$B_k^*=-\frac{2}{n_k^2}+\frac{2+\mu}{n_k}\Delta r\frac{D_k^/}{D_k}+\Delta r^2\frac{D_k^{//}}{D_k} \qquad (10)$$
$$C_k^*=\frac{1}{2n_k}\left(\frac{3}{n_k^2}-\frac{3\Delta r}{n_k}\frac{D_k^/}{D_k}+\mu\Delta r^2\frac{D_k^{//}}{D_k}\right)$$
$$E_k^*=-2\frac{C_k^/}{n_k}$$

(5) yechimni radial yo'nalishda burilish burchagi uchun ichki zo'riqish kuchlarini

ifodalovchi formularga qo'yib quyidagi natijalarga ega bo'lamiz. [1]

$$M_r = -D\left[\frac{d^2W}{dr^2} + \mu\left(\frac{1}{r}\frac{dW}{dr} - \frac{W}{r^2}\right)\right]\cos\varphi;$$

$$M_\varphi = -D\left[\mu\frac{d^2W}{dr^2} + \frac{1}{r}\frac{dW}{dr} - \frac{W}{r^2}\right]\cos\varphi;$$

$$M_{r\varphi} = D(1-\mu)\left(\frac{1}{r}\frac{dW}{dr} - \frac{W}{r^2}\right)\sin\varphi$$

$$Q_r = -\frac{dW}{dr}\cos\varphi$$

$$Q_r = \left\{-D\left(\frac{d^3W}{dr^3} + \frac{1}{r}\frac{d^2W}{dr^2} - \frac{2}{r^2}\frac{dW}{dr} + \frac{2}{r^3}W\right) - \right. \qquad (11)$$

$$\left. -\frac{dD}{dr}\left[\frac{d^2W}{dr^2} + \mu\left(\frac{1}{r}\frac{dW}{dr} - \frac{W}{r^2}\right)\right]\right\}\cos\varphi;$$

(7) va (11) formulalarni hisobga olgan holda bu ifodalarni quyidagicha chekli ayirmali ko'rinishda yozish mumkin.

$$Q_\phi = \left[D\left(\frac{1}{r}\frac{d^2W}{dr^2} + \frac{2}{r^2}\frac{dW}{dr} - \frac{W}{r^2}\right) + (1-\mu)\frac{dP}{dr}\left(\frac{1}{r}\frac{dW}{dr} - \frac{W}{r^2}\right)\right]\sin\phi;$$

$$V_r = \mp\left\{D\left(\frac{d^3W}{dr^3} + \frac{1}{r}\frac{d^2W}{dr^2} - \frac{3-\mu}{r^2}\frac{dW}{dr} + \frac{3-\mu}{r^3}W\right) + \right.$$

$$\left. + \frac{dD}{dr}\left[\frac{d^2W}{dr^2} + \mu\left(\frac{1}{r}\frac{dW}{dr} - \frac{W}{r^2}\right)\right]\right\}\cos\phi.$$

$$\theta_{r,k} = -\frac{W_l - W_i}{2\Delta r}\cos\varphi;$$

$$M_{r,k} = -\frac{D_k}{\Delta r^2}\left[W_l - 2W_k + W_i + \frac{\mu}{n_k}\left(\frac{W_l - W_i}{2} - \frac{\omega_k}{n_k}\right)\right]\cos\varphi;$$

$$M_{\varphi,k} = -\frac{D_k}{\Delta r^2}\left[\frac{1}{n_k}\left(\frac{W_l - W_i}{2} - \frac{W_k}{n_k}\right) + \mu(W_l - 2W_k + W_i)\right]\cos\varphi;$$

$$M_{r\varphi,k} = M_{\varphi r,k} = \frac{(1-\mu)D_k}{2n_k\Delta r^2}\left(W_l - W_i - \frac{2W_k}{n_k}\right)\sin\varphi;$$

$$Q_{r,k} = -\frac{D_k}{\Delta r^3}\left\{\frac{1}{2}(W_m - W_n) + W_l\left[-1 + \frac{1}{n_k} - \frac{1}{n_k^2} + \left(1 + \frac{\mu}{2n_k}\right)\Delta r\frac{D_k'}{D_k}\right]\right.$$

$$\left. + W_i\left[+1 + \frac{1}{n_k} + \frac{1}{n_k^2} + \left(1 - \frac{\mu}{2n_k}\right)\Delta r\frac{D_k'}{D_k}\right] + 2W_k\left[\frac{1}{n_k}\left(\frac{1}{n_k^2} - 1\right) - \left(1 + \frac{\mu}{2n_k^2}\right)\Delta r\frac{D_k'}{D_k}\right]\right\}\cos\varphi;$$

$$Q_{\varphi,k} = \frac{D_k}{2n_k\Delta r^3}\left\{W_l\left[2 + \frac{1}{n_k} + (1-\mu)\Delta r\frac{D_k'}{D_k}\right] - \frac{2}{n_k}W_k\left[\frac{1}{n_k} + (1-\mu)\Delta r\frac{D_k'}{D_k}\right] + \right.$$

$$\left. + W_i\left[2 - \frac{1}{n_k} - (1-\mu)\Delta r\frac{D_k'}{D_k}\right]\right\}\sin\varphi;$$

$$V_{r,k} = \mp\frac{D_k}{\Delta r^3}\left\{\frac{1}{2}(W_m - W_k) + W_l\left[-1 + \frac{1}{n_k} - \frac{3-\mu}{2n_k^2} + \left(1 + \frac{\mu}{2n_k^2}\right)\Delta r\frac{D_k'}{D_k}\right] + \right.$$

$$+ W_i\left[+1 + \frac{1}{n_k} + \frac{3-\mu}{2n_k^2} + \left(1 - \frac{\mu}{2n_k}\right)\Delta r\frac{D_k'}{D_k}\right] + \qquad \mathbf{(12)}$$

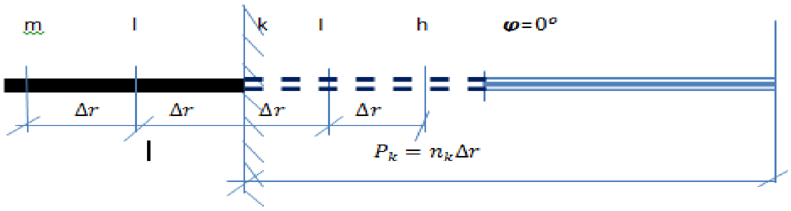
$$\left. + 2W_k\left[\frac{1}{n_k}\left(\frac{3-\mu}{2n_k^2} - 1\right) - \left(1 + \frac{\mu}{2n_k^2}\right)\Delta r\frac{D_k'}{D_k}\right]\right\}\cos\varphi$$

Qistirib mahkamlangan tomon.

Bu holda k nuqta uchun (2) qistirib mahkamlanganli shartidan $W_k = \theta_{r,k} = 0$; (8) tenglamalardan esa

$$W_e = W_i \qquad (13)$$

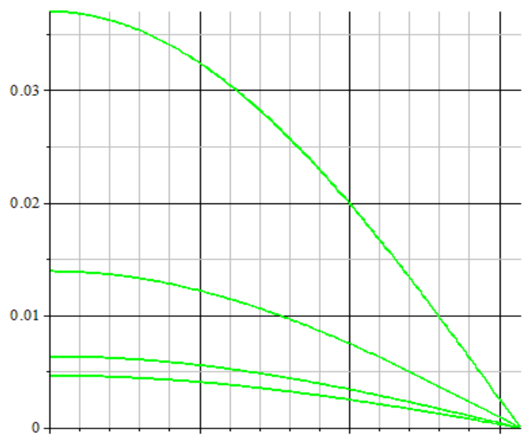
k nuqta plastinka ichki tomon yoyida joylashgan holat.

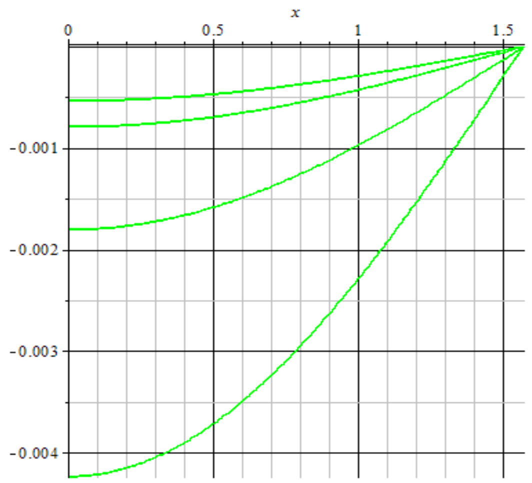


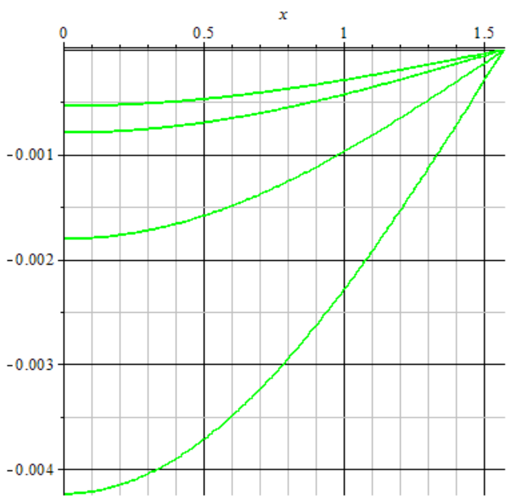
Chegaradagi nuqtadan W_m ko'chish $W_m = \frac{1}{F_m^*}\left(\frac{P_a n_k}{aD_k}\Delta r^5 - WF_e^* - W_i F_i^* - W_h F_h^*\right)$ formula orqali aniqlanadi.

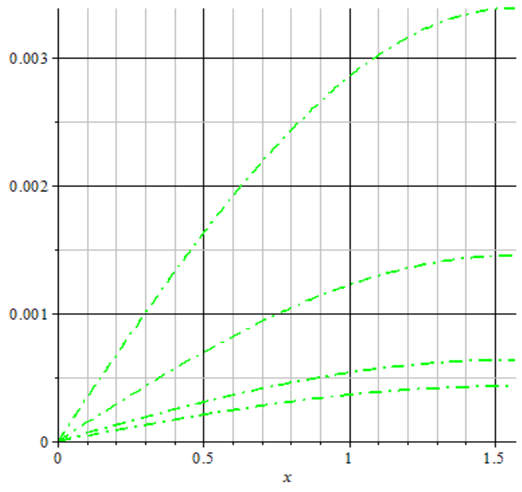
Olingan natijalar. masalaning aniq son qiymatlari **Mo:=1;a:=9; r:=4.5; r2:=6; r3:=7.5; R:=9; E:=2*10^5; dd:=1.5; Pa:=4*Mo/(a^3*pi);nu:=1/6;** yechimlar quyidagi grafiklarda tasvirlangan.

M_r radial moment grafigi plastinka turli kesimlaridagi nuqtalari φ burchakdan bog'liq o'zgarishi



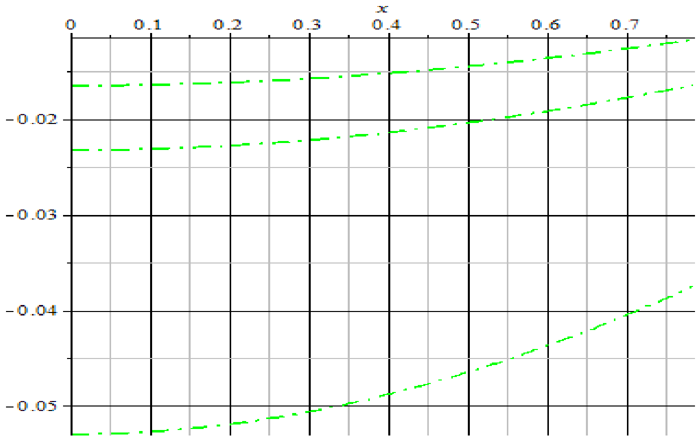






M_{φ} tangensial moment grafigi plastinka turli kesimlaridagi nuqtalari φ burchakdan bog'liq o'zgarishi

$M_{r\varphi}$ burovchi moment grafigi plastinka turli kesimlaridagi nuqtalari φ burchakdan bog'liq ozgarishi



Q Qirquvchi kuch grafigi plastinka turli kesimlarida nuqtalari φ burchakdan bog'liq o'zgarishi

Xulosa

Barcha grafiklar tahlili shuni ko'rsatadiki, plastinkaning mahkamlangan chegarasidan uzoqlashgan sari ichki zo'riqishlar va momentlar kamayadi. Bu fizik jihatdan ham to'g'ri natija bo'lib, yuklanish ta'siri asosan chegaralarda kuchliroq namoyon bo'lishini tasdiqlaydi. Chekli ayirmalar usuli esa bunday murakkab masalalarni yechishda samarali va ishonchli vosita ekanligi isbotlandi. Ushbu yondashuv yordamida muhandislik amaliyotida uchraydigan qalinligi o'zgaruvchi plastinkalar masalalarini yetarli aniqlikda hisoblash mumkin.

Foydalanilgan adabiyotlar ro'yxati:

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