SONLI QATORLARNI YAQINLASHISHGA TEKSHIRISHDA VA ANIQMAS INTEGRALLARNI HISOBLASHDA MATEMATIK INDUKSIYA METODI

B.Mamadaliyev
(fizika-matematika fanlari nomzodi, QDU),
X.Tadjimatova (pedagogika fanlari bo'yicha falsafa doktori, PhD QDU),
***S.Turonova** (magistr, QDU)*

Matematik induksiya metodi matematikaning turli – tuman, hatto bir – biridan juda olis sohalarida muvaffaqiyat bilan keng qo'llaniladigan metoddir.

Ushbu maqolada sonli qatorlarni yaqinlashishga tekshirishda va aniqmas integrallarni hisoblashda matematik induksiya metodining tadbiqlarini misollar yordamida ko'rsatishga harakat qilingan.

Ma'lumki, musbat qatorning yaqinlashuvchiligi yoki uzoqlashuvchiligini bilgan holda, hadlari bir qator hadlari bilan ma'lum munosbatada bo'lgan (taqqoslangan) ikkinchi musbat qatorning aniqlash mumkin. Ularga oid bir necha teoremalar mavjud. Bu teoremalardan birini keltiramiz.

Teorema. Agar n ning biror $n_0 (n_0 \geq 1)$ qiymatidan boshlab barcha $n \geq n_0$ lar uchun

$$a_n \leq b_n$$

tenglik o'rinli bo'lsa, u holda $\sum_{n=1}^{\infty} b_n$ qatorning yaqinlashuvchi bo'lishidan $\sum_{n=1}^{\infty} a_n$ qatorning ham yaqinlashuvchi bo'lishi yoki $\sum_{n=1}^{\infty} a_n$ qatorning uzoqlashuvchi bo'lishidan $\sum_{n=1}^{\infty} b_n$ qatorning ham uzoqlashuvchi bo'lishi kelib chiqadi.

1-misol. $\sum_{n=1}^{\infty} a_n = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{n!} + \ldots = \sum_{n=1}^{\infty} \frac{1}{n!}$ qatorni yaqinlashishga tekshiramiz.

Barcha $n \geq 1$ lar uchun

$$A(n): \frac{1}{n!} \leq \frac{1}{2^{n-1}}$$

tengsizlik o'rinli bo'ladi. Bu tengsizlikni matematik induksiya metodi bilan tekshirish mumkin:

Isboti. I. $n = 1$ da $\frac{1}{1!} = \frac{1}{2^{1-1}}$ tenglik o'rinli.

II. $n = 1$ da $\frac{1}{1!} \leq \frac{1}{2^{k-1}}$ tenglik deb faraz qilamiz va $n = k + 1$ uchun tengsizlikni tekshiramiz:

$$\frac{1}{(k+1)!}=\frac{1}{k!\,(k+1)}\le\frac{1}{k!}\cdot\frac{1}{k+1}\le\frac{1}{2^{k-1}}\cdot\frac{1}{k+1}\le\frac{1}{2^{k-1}}\cdot\frac{1}{1+1}=$$
$$=\frac{1}{2^{k-1}}\cdot\frac{1}{2}=\frac{1}{2^k}.$$

Demak, $\frac{1}{(k+1)!}\le\frac{1}{2^k}$ tengsizlik barcha n-lar uchun o‘rinli ekan.

Agar $\sum_{n=1}^{\infty}b_n=\sum_{n=1}^{\infty}\frac{1}{2^{n-1}}=\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1}$ deb olsak, bu geometrik qator $\left(q=\frac{1}{2}<1\right)$ yaqinlashuvchi. U holda yuqoridagi teoremaga ko‘ra berilgan qator $\sum_{n=1}^{\infty}\frac{1}{n!}$ - yaqinlashuvchi.

2-misol. $\sum_{n=1}^{\infty}\frac{1}{1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{n}}}$ qatorni yaqinlashishga tekshiring.

Matematik induksiya metodi bilan ko‘rsatish mumkinki,

$$A(n):1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{n}}<2\cdot\sqrt{n}\qquad (n\ne 1)$$

bo‘ladi.

Bu tengsizlikni isbotlaymiz:

I. $n=1$ da $1<2$ to‘g‘ri.

II. $n=k$ da $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{k}}<2\sqrt{k}$ ni to‘g‘ri faraz qilib, $n=k+1$ uchun to‘g‘riligini ko‘rsatamiz.

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}<2\cdot\sqrt{k}+\frac{1}{\sqrt{k+1}}=$$
$$=\frac{2\sqrt{k}\cdot\sqrt{k+1}+1}{\sqrt{k+1}}\le\frac{k+(k+1)+1}{\sqrt{k+1}}=2\sqrt{k+1}.$$

(bu yerda $(\sqrt{k+1}-k)^2=k+1-2\sqrt{k}\cdot\sqrt{k+1}+k\ge 0$ tengsizlikdan foydalandik).

Demak, $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{k+1}}\le 2\sqrt{k+1}$ tengsizlik barcha n-lar o‘rinli ekan.

Agar

$$b_n=\frac{1}{1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{n}}}$$

va

$$a_n=\frac{1}{2\sqrt{n}}$$

deb olsak,

$$\sum_{n=1}^{\infty}a_n\le\sum_{n=1}^{\infty}b_n$$

munosabatdan va umumlashgan garmonik qatorning yaqinlashuvchi – uzoqlashuvchi shartiga ko‘ra hamda yuqoridagi teoremaga asosan, berilgan qatorning uzoqlashuvchiligi kelib chiqadi.

Endi aniqmas integrallarni hisoblashda matematik induksiya metodining tatbiqlariga doir misollardan namunalar keltiramiz.

3-misol.

$$A(n):\underbrace{\int\int...\int}_{n} a^x dx=\frac{1}{\ln^n a}a^x+\sum_{i=1}^{n}\frac{c_i}{(n-i)!}x^{n-i}$$

tenglikni isbotlaymiz, bu yerda $c_1,c_2,...,c_n$ lar haqiqiy sonlar.

Isboti. Bu tenglikni matematik induksiya metodidan foydalanib isbotlaymiz.

I. $n=1$ da $\int a^x dx=\frac{1}{1_n^k}\cdot a^x+c_1$ to‘g‘ri.

II. $n=k$ da $\underbrace{\int\int...\int}_{k} a^x dx=\frac{1}{\ln^n a}a^x+\sum_{i=1}^{n}\frac{c_i}{(k-i)!}x^{n-i}$

tenglikni o‘rinli deb faraz qilib $n=k+1$ uchun tenglikni o‘rinli bo‘lishini ko‘rsatamiz.

$$\underbrace{\int\int...\int}_{k+1} a^x dx=\underbrace{\int\int...\int}_{k} a^x dx=\int\left(\frac{1}{1_n^{k+1} a}a^x+\sum_{i=1}^{n}\frac{c_i}{(k-i)!}\cdot x^{n-i}\right)dx=$$

$$=\frac{1}{1_n^n a}\cdot a^x+\sum_{i=1}^{k}\frac{c_i}{(k-i)!}\cdot x^{k+1-i}+c_{k+1}=$$

$$=\frac{1}{1_n^{k+1} a}\cdot a^x+\sum_{i=1}^{k+1}\frac{c_i}{(k+1-i)!}\cdot x^{k+1-i}+c_{k+1}=$$

$$=\frac{1}{1_n^{k+1} a}\cdot a^x+\sum_{i=1}^{k+1}\frac{c_i}{(k+1-i)!}\cdot x^{k+1-i}.$$

Demak,

$$\underbrace{\int\int...\int}_{k+1} a^x dx=\frac{1}{1_n^{k+1} a}a^x+\sum_{i=1}^{n}\frac{c_i}{(k-i)!}\cdot x^{n-i}$$

tenglik barcha n lar uchun o‘rinlidir.

4-misol. $A(n):\underbrace{\int\int...\int}_{n} \sin xdx=\sin\left(x+n\cdot\frac{3\pi}{2}\right)+\sum_{i=1}^{k}\frac{c_i}{(k-i)!}\cdot x^{n-i}$

tenglikni isbotlang, bu yerda $c_1,c_2,...,c_n$-lar haqiqiy sonlar.

Bu tenglikni ham matematik induksiya metodi orqali isbotlaymiz.

I. $n=1$ da $\int \sin xdx=-\cos+c_1=\sin\left(x+\frac{3\pi}{2}\right)+c_1$ to‘g‘ri.

II. $n=k$ da $\underbrace{\int\int...\int}_{k} \sin xdx=\sin\left(x+n\cdot\frac{3\pi}{2}\right)+\sum_{i=1}^{k}\frac{c_i}{(k-i)!}\cdot x^{n-i}$ tenglikni to‘g‘ri deb faraz qilib, $n=k+1$ to‘g‘riligini tekshiramiz.

$$\underbrace{\int\int\ldots\int}_{k+1}\sin xdx=\underbrace{\int\int\ldots\int}_{k}\sin xdx=$$

$$=\int\left(\sin\left(x+k\frac{3\pi}{2}\right)+\sum_{i=1}^{k}\frac{c_1}{(k-i)!}\cdot x^{k+1}\right)dx=$$

$$=-\cos\left(x+k\cdot\frac{3\pi}{2}\right)+\sum_{i=1}^{k}\frac{c_1}{(k-i)!}\cdot\frac{1}{k+i+1}x^{k-i+1}+c_{k+1}=$$

$$=\sin\left(x+k\frac{3\pi}{2}+\frac{3\pi}{2}\right)+\sum_{i=1}^{k}\frac{c_1}{(k+1-i)!}x^{k-i+1}+c_{k+1}=$$

$$=\sin\left(x+(k+1)\cdot\frac{3\pi}{2}\right)+\sum_{i=1}^{k+1}\frac{c_1}{(k+1-i)!}x^{k-i+1}.$$

Demak, $\underbrace{\int\int\ldots\int}_{k+1}\sin xdx=\sin\left(x+(k+1)\cdot\frac{3\pi}{2}\right)+\sum_{i=1}^{k+1}\frac{c_i}{(k+1-i)!}\cdot x^{k+1-i}$ barcha n lar uchun o‘rinlidir.

Bunday misollarni ko‘plab keltirish mumkin.

Foydalanilgan adabiyotlar

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