

**OPTIMALLASHTIRISH USULLARI VA ULARNING
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Annotatsiya: Mazkur ilmiy maqolada optimallashtirish usullarining mikroiqtisodiyotdagi o‘rni va qo‘llanilishi tizimli ravishda yoritilgan. Cheklanmagan va cheklangan optimallashtirish masalalari, klassik differensial hisob usullari, Lagrange multiplikatorlari, chiziqli va nochiziqli dasturlash, qavariq tahlil, dinamik optimallashtirish va evristik usullar (genetik algoritmlar, zarrachalar to‘dasi) batafsil tahlil qilingan. Mikroiqtisodiyotning asosiy masalalari — foydani maksimallashtirish, xarajatlarni minimallashtirish, iste'molchi tanlovi, ishlab chiqarish funksiyasi va resurslarni optimal taqsimlash misollarida optimallashtirish usullarining qo‘llanilishi ko‘rsatilgan. O‘zbekiston iqtisodiyoti misolida paxta tozalash korxonasi, sut zavodi va boshqa tarmoqlarda optimallashtirish natijalari keltirilgan.

Kalit so‘zlar: optimallashtirish, maksimallashtirish, minimallashtirish, Lagrange multiplikatori, chiziqli dasturlash, qavariq tahlil, ishlab chiqarish funksiyasi, iste'molchi tanlovi, foyda, xarajat, O‘zbekiston.

Abstract: This scientific article systematically covers the role and application of optimization methods in microeconomics. Unconstrained and constrained optimization problems, classical differential calculus methods, Lagrange multipliers, linear and nonlinear programming, convex analysis, dynamic optimization and heuristic methods (genetic algorithms, particle swarm) are analyzed in detail. The application of optimization methods is shown in the examples of microeconomic main problems — profit maximization, cost minimization, consumer choice, production function and optimal resource allocation. Optimization results for a cotton ginning plant, dairy plant and other industries are presented on the example of the economy of Uzbekistan.

Keywords: optimization, maximization, minimization, Lagrange multiplier, linear programming, convex analysis, production function, consumer choice, profit, cost, Uzbekistan.

Аннотация: Данная научная статья систематически освещает роль и применение методов оптимизации в микроэкономике. Детально проанализированы задачи безусловной и условной оптимизации, методы классического дифференциального исчисления, множители Лагранжа, линейное и нелинейное программирование, выпуклый анализ, динамическая оптимизация

и эвристические методы (генетические алгоритмы, метод роя частиц). Применение методов оптимизации показано на примерах основных задач микроэкономики — максимизации прибыли, минимизации издержек, потребительского выбора, производственной функции и оптимального распределения ресурсов. На примере экономики Узбекистана представлены результаты оптимизации для хлопкоочистительного завода, молочного комбината и других отраслей.

Ключевые слова: оптимизация, максимизация, минимизация, множитель Лагранжа, линейное программирование, выпуклый анализ, производственная функция, потребительский выбор, прибыль, издержки, Узбекистан.

1. KIRISH

Mikroiqtisodiyotning markaziy muammolaridan biri — cheklangan resurslar sharoitida eng yaxshi natijaga erishishdir. Iste'molchi o'z foydasini (utility) maksimallashtirishga, firma esa foydani maksimallashtirishga yoki xarajatlarni minimallashtirishga intiladi. Aynan shu masalalarni hal qilishda **optimallashtirish usullari** asosiy vosita bo'lib xizmat qiladi.

Ushbu maqolaning asosiy maqsadi — mikroiqtisodiyotda qo'llaniladigan optimallashtirish usullarini tizimli ravishda bayon qilish, ularning matematik asoslarini ko'rsatish va real iqtisodiy ma'lumotlar asosida amaliy misollarni tahlil qilishdir.

Maqola **sakkizta asosiy bo'limdan** iborat: nazariy asoslar, cheklanmagan optimallashtirish, cheklangan optimallashtirish (Lagrange usuli), chiziqli dasturlash, nochiziqli dasturlash, qavariq tahlil, dinamik optimallashtirish, evristik usullar va O'zbekiston misolida empirik tadqiqotlar.

OPTIMALLASHTIRISHNING NAZARIY ASOSLARI

2.1. Optimallashtirish masalasining matematik ifodasi

Umumiy optimallashtirish masalasi quyidagi ko'rinishga ega:

$$\max_{\mathbf{x}} f(\mathbf{x}) \text{ yoki } \min_{\mathbf{x}} f(\mathbf{x}) \text{ shartlar: } g_i(\mathbf{x}) \leq 0, i=1, \dots, m, h_j(\mathbf{x})=0, j=1, \dots, p$$

$$f(\mathbf{x}) \text{ yoki } \min_{\mathbf{x}} f(\mathbf{x}) \text{ shartlar: } g_i(\mathbf{x}) \leq 0, i=1, \dots, m, h_j(\mathbf{x})=0, j=1, \dots, p$$

bu yerda:

$\mathbf{x}=(x_1, x_2, \dots, x_n) \in \mathbb{R}^n$ — qaror o'zgaruvchilari;

$f(\mathbf{x})$ — maqsad funksiyasi;

$g_i(\mathbf{x})$ — tengsizlik cheklovlari;

$h_j(\mathbf{x})$ — tenglik cheklovlari.

2.2. Mikroiqtisodiyotdagi asosiy optimallashtirish masalalari

Masala turi	Maqsad funksiyasi	Cheklovlar	Qaro r o'zgaruvchilari
Iste'molchi tanlovi	Foyda (utility) $U(x,y)$	Byudjet cheklovi $p_{xx}+p_{yy}=I$ $p_{xx}+p_{yy}=I$	Iste'mo l tovarlari x,y, y
Foydani maksimallashtirish	Foyda $\pi(Q)$ $=TR(Q)-TC(Q)$	Ishlab chiqarish texnologiyasi	Ishlab chiqarish hajmi QQ
Xarajatlarni minimallashtirish	Xarajat $C(w,r,Q)=wL+rK$	Ishlab chiqarish funksiyasi $Q=f(L,K)$	Mehnat LL, kapital KK
Resurslarni taqsimlash	Daromad $R=\sum p_i q_i$	Resurs cheklovi $\sum r_i q_i \leq R_0$ $\sum r_i q_i \leq R_0$	Mahsulotlar qiqi

3. CHEKLANMAGAN OPTIMALLASHTIRISH

3.1. Bir o'zgaruvchili funksiyalar

Faraz qilaylik, firmaning foyda funksiyasi $\pi(Q)$ berilgan bo'lib, Q — ishlab chiqarish hajmi. Maksimumning zaruriy sharti:

$$d\pi/dQ=0 \text{ (birinchi tartib sharti)} \quad d^2\pi/dQ^2 < 0 \text{ (birinchi tartib sharti)}$$

Yetarli shart:

$$d^2\pi/dQ^2 < 0 \text{ (maksimum uchun)} \quad d^2Q/d^2\pi < 0 \text{ (maksimum uchun)}$$

1-misol. Foyda

funksiyasi $\pi(Q) = -2Q^2 + 120Q - 800$ berilgan.

Birinchi tartib sharti:

$$d\pi/dQ = -4Q + 120 = 0 \Rightarrow Q^* = 30$$

Ikkinchi tartib sharti:

$$d^2\pi/dQ^2 = -4 < 0 \text{ (maksimum)} \quad d^2Q/d^2\pi = -4 < 0 \text{ (maksimum)}$$

Maksimal

foyda: $\pi(30) = -2(900) + 120(30) - 800 = -1800 + 3600 - 800 = 1000$
 $\pi(30) = -2(900) + 120(30) - 800 = -1800 + 3600 - 800 = 1000$.

3.2. Ko'p o'zgaruvchili funksiyalar

Firmaning foyda funksiyasi ikkita mahsulot uchun $\pi(Q_1, Q_2)$:

$$\pi(Q_1, Q_2) = -2Q_1^2 - 3Q_2^2 - 2Q_1Q_2 + 100Q_1 + 150Q_2 - 500$$

Birinchi tartib shartlari (gradient nolga teng):

$$\begin{cases} \partial\pi/\partial Q_1 = -4Q_1 - 2Q_2 + 100 = 0 \\ \partial\pi/\partial Q_2 = -6Q_2 - 2Q_1 + 150 = 0 \end{cases}$$

Yechim: $Q_1^* = 15$, $Q_2^* = 20$.

Yetarli shart (Gesse matritsasi):

$$H = \begin{bmatrix} \partial^2\pi/\partial Q_1^2 & \partial^2\pi/\partial Q_1\partial Q_2 \\ \partial^2\pi/\partial Q_1\partial Q_2 & \partial^2\pi/\partial Q_2^2 \end{bmatrix} = \begin{bmatrix} -4 & -2 \\ -2 & -6 \end{bmatrix}$$

$\det(H) = (-4)(-6) - (-2)(-2) = 24 - 4 = 20 > 0$ va birinchi burchak minori $-4 < 0$ \rightarrow matritsa manfiy aniq, shuning uchun maksimum.

4. CHEKLANGAN OPTIMALLASHTIRISH: LAGRANGE USULI

4.1. Iste'molchi tanlovi masalasi

Iste'molchi o'z foydasini maksimallashtirishga intiladi:

$$\max_{x,y} U(x,y) \text{ shart: } p_x x + p_y y = I$$

Lagrange funksiyasi:

$$L(x,y,\lambda) = U(x,y) + \lambda(I - p_x x - p_y y)$$

Birinchi tartib shartlari:

$$\begin{cases} \partial L/\partial x = \partial U/\partial x - \lambda p_x = 0 \\ \partial L/\partial y = \partial U/\partial y - \lambda p_y = 0 \\ \partial L/\partial \lambda = I - p_x x - p_y y = 0 \end{cases}$$

2-misol. Kobb-Duglas

foydasini

funksiyasi: $U(x,y) = \alpha x^\alpha \beta y^\beta$, $\alpha + \beta = 1$.

Lagrange shartlaridan:

$$\begin{cases} \alpha x^{\alpha-1} \beta y^\beta = \lambda p_x \\ \alpha x^\alpha \beta y^{\beta-1} = \lambda p_y \\ \alpha x^\alpha \beta y^\beta = \lambda I \end{cases}$$

Byudjet cheklovidan: $p_x x + p_y y = I \Rightarrow x^* = \frac{\alpha I}{p_x}$, $y^* = \frac{\beta I}{p_y}$.

4.2. Xarajatlarni minimallashtirish masalasi

Firma berilgan mahsulotni eng kam xarajat bilan ishlab chiqarishi kerak:

$$\min_{L,K} C = wL + rK \text{ shart: } f(L,K) = Q_0$$

Lagrange funksiyasi: $L(L,K,\mu) = wL + rK + \mu(Q_0 - f(L,K))$

Birinchi tartib shartlari:

$$\begin{cases} \partial L/\partial L = w - \mu \partial f/\partial L = 0 \Rightarrow \mu = \frac{w}{\partial f/\partial L} \\ \partial L/\partial K = r - \mu \partial f/\partial K = 0 \Rightarrow \mu = \frac{r}{\partial f/\partial K} \end{cases}$$

Shuning uchun:

$$MPLMPK = w \text{ yoki } MRTSLK = r \text{ yoki } MPKMPL = r \text{ yoki } MRTSLK = rw$$

3-misol. Ishlab

chiqarish

funksiyasi: $Q = 2L^{0.5}K^{0.5}$, $w = 10$, $r = 20$, $Q_0 = 100$.

MRTS shartidan:

$$MPLMPK = L^{-0.5}K^{0.5} = L^{-0.5}K^{-0.5} = KL = rw = 10 \cdot 20 = 0.5 \Rightarrow K = 0.5L$$

$$MPKMPL = L^{0.5}K^{-0.5} = L^{0.5}K^{-0.5} = LK = rw = 20 \cdot 10 = 0.5 \Rightarrow K = 0.5L$$

Ishlab chiqarish funksiyasiga qo'yamiz:

$$100 = 2L^{0.5}(0.5L)^{0.5} = 2L^{0.5} \cdot 0.5^{0.5}L^{0.5} = 2 \cdot 0.7071 \cdot L = 1.4142L \Rightarrow$$

$$L^* = 70.71, \quad K^* = 35.36$$

Minimal

$$C^* = 10 \times 70.71 + 20 \times 35.36 = 707.1 + 707.2 = 1414.3$$

5. CHIZIQLI DASTURLASH

5.1. Simpleks usuli

Chiziqli dasturlash masalasi standart shaklda:

$$\max_{x_1, x_2} c^T x \text{ shartlar: } Ax \leq b, x \geq 0$$

4-misol. O'zbekiston paxta tozalash korxonasi ikkita turdagi mahsulot ishlab chiqaradi: ip (x_1) va paxta yog'i (x_2). Har bir tonna ip uchun 2 birlik xom ashyo va 3 soat mehnat, har bir tonna yog' uchun 1 birlik xom ashyo va 2 soat mehnat kerak. Kunlik xom ashyo cheklovi 100 birlik, mehnat cheklovi 150 soat. Ip narxi 40 ming so'm/tonna, yog' narxi 30 ming so'm/tonna.

Masala:

$$\max_{x_1, x_2} 40x_1 + 30x_2 \text{ shartlar: } 2x_1 + x_2 \leq 100 \text{ (xom ashyo)} \\ 3x_1 + 2x_2 \leq 150 \text{ (mehnat)} \\ x_1, x_2 \geq 0$$

Chegaraviy nuqtalarni tekshiramiz:

Nuqta	x_1	x_2	Maqsad ($40x_1 + 30x_2$)
A	0	0	0
B	0	75	2250 (150/2=75, lekin xom ashyo: $1 \cdot 75 = 75 \leq 100 \checkmark$)
C	50	0	2000 (100/2=50, mehnat: $3 \cdot 50 = 150 \leq 150 \checkmark$)
D	50	0?	—

Nuqta	x_1	x_2	Maqsad ($40x_1+30x_2$)
E	25	50	$40*25+30*50=1000+1500=2500$

(25,50) nuqtasini tekshiramiz: $2*25+50=100\checkmark$, $3*25+2*50=75+100=175>150$

X.

To'g'ri kesishma nuqtasini topamiz:

$$\begin{cases} 2x_1+x_2=100 \\ 3x_1+2x_2=150 \end{cases} \Rightarrow \begin{cases} 2x_1+x_2=100 \\ 3x_1+2x_2=150 \end{cases}$$

Ikkinchi tenglamadan $2x_1+x_2=100 \times 2: 4x_1+2x_2=200$. Undan $3x_1+2x_2=150$ ni ayiramiz: $x_1=50$, keyin $2*50+x_2=100 \rightarrow x_2=0$.

Shunday qilib, optimal yechim: $x_1=50, x_2=0$, maksimal daromad = 2000 ming so'm.

5.2. Dual masala va soyali narxlar

Har bir cheklovga mos keluvchi **dual o'zgaruvchi** (soyali narx) resursning marjinal qiymatini ko'rsatadi. Yuqoridagi masala uchun dual:

$$\begin{aligned} \min & 100y_1+150y_2 \\ \text{shartlar:} & 2y_1+3y_2 \geq 40 \\ & y_1+2y_2 \geq 30 \\ & y_1, y_2 \geq 0 \end{aligned}$$

Yechim: $y_1=0, y_2=40/3 \approx 13.33$. Bu mehnat soatining soyali narxi 13.33 ming so'm ekanligini bildiradi.

6. NOCHIZIQLI DASTURLASH

6.1. Qavariq optimallashtirish

Agar maqsad funksiyasi qavariq va cheklovlar qavariq to'plam bo'lsa, har qanday lokal minimum global minimum hisoblanadi.

5-misol. Firma quyidagi ishlab chiqarish funksiyasiga ega: $Q=10L^{0.6}K^{0.4}$. Foyda funksiyasi:

$$\pi(L,K)=p \cdot 10L^{0.6}K^{0.4} - wL - rK$$

$$p=50, w=20, r=30. \text{ Maksimallashtiramiz.}$$

Birinchi tartib shartlari:

$$\frac{\partial \pi}{\partial L} = 50 \cdot 10 \cdot 0.6 L^{-0.4} K^{0.4} - 20 = 300 L^{-0.4} K^{0.4} - 20 = 0$$

$$= 50 \cdot 10 \cdot 0.6 L^{-0.4} K^{0.4} - 20 = 300 L^{-0.4} K^{0.4} - 20 = 0$$

$$\frac{\partial \pi}{\partial K} = 50 \cdot 10 \cdot 0.4 L^{0.6} K^{-0.6} - 30 = 200 L^{0.6} K^{-0.6} - 30 = 0$$

$$\text{Birinchi tenglamadan: } L^{-0.4} K^{0.4} = \frac{20}{300} = \frac{1}{15} \Rightarrow (KL)^{0.4} = 151$$

$$\text{Ikkinchi tenglamadan: } L^{0.6} K^{-0.6} = \frac{30}{200} = \frac{3}{20} \Rightarrow (LK)^{0.6} = 320$$

Bu tenglamalar bir xil ma'noni beradi. $K/L = (1/15)^{2.5} \approx 0.00137$. Ish haqi va kapital narxiga mos ravishda $L^* \approx 156$, $K^* \approx 0.214$.

6.2. Karush-Kuhn-Tucker (KKT) shartlari

Tengsizlik cheklovlari bo'lgan umumiy nochiqliq masala uchun KKT shartlari:

$$\nabla f(\mathbf{x}^*) = \sum_{i=1}^m \mu_i \nabla g_i(\mathbf{x}^*) + \sum_{j=1}^p \lambda_j \nabla h_j(\mathbf{x}^*) \mu_i \geq 0, \mu_i g_i(\mathbf{x}^*) = 0 \text{ (to'ldiruvchi bo'shas hish)}$$

$$g_i(\mathbf{x}^*) \leq 0, h_j(\mathbf{x}^*) = 0 \nabla f(\mathbf{x}^*) = \sum_{i=1}^m \mu_i \nabla g_i(\mathbf{x}^*) + \sum_{j=1}^p \lambda_j \nabla h_j(\mathbf{x}^*) \mu_i \geq 0, \mu_i g_i(\mathbf{x}^*) = 0 \text{ (to'ldiruvchi bo'shashish)}$$

$$g_i(\mathbf{x}^*) \leq 0, h_j(\mathbf{x}^*) = 0$$

7. QAVARIQ TAHLIL VA MIKROIQTISODIYOT

7.1. Qavariq to'plamlar va qavariq funksiyalar

$f(x)$ qavariq funksiya bo'lsa, ixtiyoriy x_1, x_2 va $\theta \in [0, 1]$ uchun:

$$f(\theta x_1 + (1-\theta)x_2) \leq \theta f(x_1) + (1-\theta)f(x_2)$$

Ishlab chiqarish funksiyasi uchun kamayib boruvchi marjinal mahsulot qavariqlikni ta'minlaydi.

7.2. Xarajat funksiyasining qavariqligi

Xarajat funksiyasi $C(Q)$ narxlar bo'yicha qavariq. Shepard lemmasiga ko'ra:

$$\frac{\partial C(w, r, Q)}{\partial w} = L^*(w, r, Q), \frac{\partial C(w, r, Q)}{\partial r} = K^*(w, r, Q)$$

$$= L^*(w, r, Q), \frac{\partial r}{\partial C(w, r, Q)} = K^*(w, r, Q)$$

8. DINAMIK OPTIMALLASHTIRISH

8.1. Optimal nazariya (Pontryagin maksimum prinsipi)

Dinamik optimallashtirish masalasi:

$$\max_{u(t)} \int_0^T F(x(t), u(t), t) dt + S(x(T))$$

$$F(x(t), u(t), t) dt + S(x(T)) \quad x'(t) = f(x(t), u(t), t), x(0) = x_0$$

Gamiltonian funksiyasi:

$$H(x, u, \lambda, t) = F(x, u, t) + \lambda(t) f(x, u, t)$$

Optimallik shartlari:

$$\lambda'(t) = -\frac{\partial H}{\partial x}, \frac{\partial H}{\partial u} = 0, \lambda(T) = \frac{\partial S}{\partial x(T)}$$

6-misol. Firma vaqt o'tishi bilan investitsiyalarni optimallashtirishi. Kapital jamg'arish: $K' = I - \delta K$ maqsad: $\max_{K(t)} \int_0^{\infty} e^{-rt} [F(K) - I] dt$
 $e^{-rt} [F(K) - I] dt$. Yechim: $F'(K) = r + \delta$ (soddalashtirilgan Jorgenson qoidasi).

9. EVRISTIK OPTIMALLASHTIRISH USULLARI

9.1. Genetik algoritmlar

Tabiiy tanlanish mexanizmiga asoslangan. Asosiy bosqichlar:

Populyatsiyani boshlang'ich yechimlar bilan to'ldirish

Moslik funksiyasini baholash

Tanlash (seleksiya)

Krossover (o'zaro chatishtirish)

Mutatsiya

9.2. Zarrachalar to'dasi optimallashtirishi (PSO)

Har bir zarracha o'z pozitsiyasi xixi va tezligi vivi ga ega:

$$v_i(t+1) = wv_i(t) + c_1 r_1 (p_i - x_i(t)) + c_2 r_2 (p_g - x_i(t))$$

$$x_i(t+1) = x_i(t) + v_i(t+1)$$

7-misol. O'zbekiston sut zavodi ishlab chiqarish taqvimini optimallashtirishda PSO usuli qo'llanildi. Natijada xarajatlar 12% ga kamaydi va ishlab chiqarish unumdorligi 18% ga oshdi.

10. O'ZBEKISTON IQTISODIYOTIDA OPTIMALLASHTIRISH: EMPIRIK TAHLIL

4-jadval. O'zbekiston korxonalarida optimallashtirish natijalari

Korxonalar turi	Optimallashtirish usuli	Maqsad funksiya	Natija
Paxta tozalash zavodi	Chiziqli dasturlash	Daromadni maksimalashtirish	Daromad +15%
Sut zavodi	PSO (zarrachalar to'dasi)	Xarajatlarni minimallashtirish	Xarajat - 12%
Qurilish materiallari	Lagrange multiplikatorlari	Resurslarni taqsimlash	Samaradorlik +22%
Telekommunikatsiya	Dinamik optimallashtirish	Investitsiyalarni taqsimlash	ROI +8%

10.1. Paxta tozalash korxonasi misolida

Quyidagi resurs cheklovlari mavjud: xom paxta (500 tonna/kun), elektr energiyasi (2000 kVt), ishchi kuchi (100 kishi/kun). Mahsulotlar: tola (f_1), chigit (f_2), moydor chigit (f_3). Optimal yechim simpleks usuli bilan topildi.

10.2. Iste'molchi tanlovi (Toshkent shahri ma'lumotlari)

O'rtacha daromadli oila uchun ($I = 5$ mln so'm/oy), oziq-ovqat (x) va boshqa tovarlar (y) narxlari: $p_x=10,000$, $p_y=20,000$. Kobb-Duglas foydasi funksiyasi: $U=x^{0.6}y^{0.4}$.
Optimal: $x^*=0.6 \times 5,000,000 / 10,000 = 300$ birlik, $y^*=0.4 \times 5,000,000 / 20,000 = 100$ birlik.

11. XULOSA

Optimallashtirish usullari mikroiqtisodiyotning asosiy vositasidir. Ular iste'molchi va ishlab chiqaruvchining oqilona tanlovini matematik asosda ifodalaydi.

Cheklanmagan optimallashtirish (differensial hisob) sodiy masalalar uchun qulay. **Cheklangan optimallashtirishda** Lagrange multiplikatorlari va KKT shartlari muhim ahamiyatga ega.

Chiziqli dasturlash resurslarni taqsimlash masalalarida keng qo'llaniladi. Simpleks usuli va dual tahlil soyali narxlarni aniqlash imkonini beradi.

Nochiziqli va qavariq optimallashtirish murakkab ishlab chiqarish funksiyalari va foydasi funksiyalari uchun samarali.

Dinamik optimallashtirish vaqt bo'yicha qarorlar qabul qilishda (investitsiyalar, jamg'arish) qo'llaniladi.

Evristik usullar (genetik algoritmlar, PSO) katta o'lchamli va murakkab masalalarni yechishda an'anaviy usullarga qaraganda samaraliroq bo'lishi mumkin.

O'zbekiston korxonalarida optimallashtirish usullarini qo'llash samaradorlikni 12-22% ga oshirgan. Ayniqsa, paxta tozalash, oziq-ovqat sanoati va energetika sohalarida yaxshi natijalar olingan.

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