HILFER OPERATORLI KASRLI DIFFUZIYA TENGLAMASI UCHUN TESKARI MASALALAR

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Annotatsiya. Uhbu maqolada vaqt bo'yicha Hilfer operatorli kasrli diffuziya tenglamasi uchun teskari masalalar ko'rib chiqiladi. To'g'ri masala bu tenglama uchun Koshi tipidagi boshlang'ich shartlar va Drixle chegaraviy shartlar bilan berilgan boshlang'ich-chegaraviy masaladir. Keyin, to'g'ri masalaning yechimiga qo'shimcha shartlar qo'yish orqali tenglamadagi noma'lum koeffitsiyentlarni aniqlashning teskari masalasini o'rganamiz. Vaqtga bog'liq koeffitsiyentni aniqlashning teskari masalasi Volterra tipidagi ekvivalent integral tenglamaga keltiriladi. Yechimning mavjudligi va yagonaligini isbotlash uchun qisqaruvchi akslantirishlar prinsipiga tayaniladi. Teskari masalaning yechimining lokal mavjudligi va global yagonaligi isbotlangan. Keyin fazoviy o'zgaruvchiga bog'liq koeffitsiyentni aniqlash bo'yicha teskari masalan yagona yechimining mavjudligi isbotlangan.

Kalit so'zlar: Rimann-Liuvill kasr integrali, Rimann-Liuvill kasr hosilasi, to'g'ri masala, teskari masala, integral tenglama, Furye qatori, mavjudlik va yagonalik, Banaxning qo'zg'almas nuqta haqidagi teoremasi.

ОБРАТНЫЕ ЗАДАЧИ ДЛЯ УРАВНЕНИЯ ДРОБНОЙ ДИФФУЗИИ С ОПЕРАТОРОМ ХИЛЬФЕРА

Аннотация. В данной статье рассматриваются обратные задачи для уравнения дробной диффузии с оператором Хильфера по времени. Прямой задачей является начально-краевая за- дача для этого уравнения с начальными данными типа Коши и граничными условиями Дирихле. Далее изучаем обратную задачу определения неизвестных коэффициентов из уравнения путем добавления дополнительных условий к решению прямой задачи. Обратная задача определения коэффициента зависящего от времени, сводится к эквивалентному интегральному уравнению типа Вольтерра. Для доказательство существования и единственности решения используется принцип сжимающих отображений. Доказаны локальное существование и глобальная единственность решения обратной задачи. Затем доказано существование един- ственного решения обратной задачи по определению коэффициента, зависящего от пространственной переменной.

Ключевые слова: дробный интеграл Римана-Лиувилля, дробная производная Римана-Лиувилля, прямая задача, обратная задача, интегральное уравнение, ряд Фурье, существование, един- ственность, теорема Банаха о неподвижной точке.

INVERSE PROBLEMS FOR THE FRACTIONAL DIFFUSION EQUATION WITH THE HILFER OPERATOR.

Abstract. This article considers inverse problems for the fractional diffusion equation with the Hilfer operator. In time. The direct problem is an initial boundary value problem for this equation with Cauchy-type initial data and Drichlet boundary conditions. We then study the inverse problem of determining unknown coefficients from the equation by adding additional conditions to the solution of the direct problem. The inverse problem of determining the time-dependent coefficient is reduced to an equivalent Volterra-type integral equation. The principle of contraction mappings is used to prove the existence and uniqueness of the solution. The local existence and global uniqueness of the solution to the inverse problem are proved. Then, the existence of a unique solution to the inverse problem of determining the coefficient depending on the spatial variable is proved.

Keywords: Riemann-Liouville fractional integral, Riemann-Liouville fractional derivative, direct problem, inverse problem, integral equation, Fourier series, existence, uniqueness, Banach fixed-point theorem.

So'nggi o'n yilliklarda nolokal dinamik jarayonlarni tasvirlash uchun kasr operatorlari bo'yicha adabiyotlar soni sezilarli darajada oshdi. Kasr hosilalar fizika, biologiya, iqtisodiyot, texnika fanlari va boshqalarda turli jarayonlarni modellashtirishda tobora ko'proq qo'llanilmoqda. [1]-[4]. Klassik kasr hosilalaridan tashqari, turli vaziyatlarda modellarning haqiqatga yaxshiroq mos kelishi uchun bir nechta umumlashmalar kiritildi. Kasr tartibli differensial tenglamalar matematik nazariyasiga kelsak, oddiy differensial tenglamalar uchun hozirgi vaziyat xususiy hosilali differensial tengla malar uchun vaziyatdan farq qiladi. Kasr diffuziya tenglamalari uchun to'g'ridan to'g'ri masalalar, masalan, boshlang'ich yoki chegaraviy masalalar [5]-[7] ishlarida keng o'rganilgan. [8],[9] ishlarda vaqt o'zgaruvchisi bo'yicha regulyarlash tirilgan kasr hosilasi va fazoviy o'zgaruvchilarda ishlaydigan o'zgaruvchan koeffitsiyentli bir tekis elliptik operatorli evolyutsion tenglama ko'rib chiqildi. Koshi masalasining fundamental yechimi qurildi va o'rganildi. Teskari masalalarda, kasr tartibli differensial tenglamalar uchun ko'pincha fazoviy o'zgaruvchilarga yoki vaqtga bog'liq bo'lgan manbani aniqlash masalala ri o'rganiladi. [10]-[15] maqolalarida kasr diffuziya-to'lqin tenglamasi uchun kichik hadi oldidagi koeffitsiyentni aniqlash bo'yicha teskari masalalar o'rganilgan. Ushbu tadqiqotning asosiy natijalari yechimning mavjudligi va yagonaligi teoremalari, shuningdek, yechimning turg'unlik bahosidir. Integro-differensial tenglamaning integral hadidagi k(t) yadrosini aniqlash masalalari ko'plab nashrlarda o'rganilgan[16]-[28], ularda klassik boshlang'ich, boshlang'ich-chegaraviy shartlari bilan bir o'lchovli va ko'p o'lchovli teskari masalalar tadqiq qilingan. Teskari masala yechimining mavjudligi va yagonaligi teoremalari isbotlangan.

Masalaning qo'yilishi

$\Omega = \{(x, t) : 0 < x < l, 0 < t \leq T\}$ sohada quyidagi kasr-differensial tenglamani qaraymiz.

$$(D_{0+}^{\alpha,\beta} u)(x,t) - u_{xx} + q(t)u(x,t) = p(x)f(t), \quad (0 < \alpha < 1;\ 0 \leq \beta \leq 1) \qquad (1)$$

boshlang'ich shart:

$$I_{0+,t}^{(1-\alpha)(1-\beta)} u(x,t)|_{t=0} = \varphi(x),\ x \in [0, l] \qquad (2)$$

chegaraviy shartlar:

$$u(0,t) = 0,\ u(l,t) = 0,\ 0 \leq t \leq T, \qquad (3)$$

$0 < \alpha < 1$ tartibli va $0 \leq \beta \leq 1$ tipli umumlashgan Rimann-Liuvill kasr differensial operatori(Hilfer) $D_{0+}^{\alpha,\beta}$ quyidagicha aniqlangan:

$$D_{0+}^{\alpha,\beta} u(\cdot, t) = \left(I_{0+,t}^{\beta(1-\alpha)} \frac{\partial^2}{\partial t^2}\left(I_{0+,t}^{(1-\beta)(1-\alpha)} u\right)\right)(\cdot, t),$$

$$I_{0+,t}^{\gamma} u(x,t) = \frac{1}{\Gamma(\gamma)} \int_0^t \frac{u(x,\tau)}{(t-\tau)^{1-\gamma}} d\tau,\ \gamma \epsilon (0,1).$$

$I_{0+,t}^{\gamma}$ — u(x, t) funksiyadan t bo'yicha olingan Rimann-Liuvill kasr integral,$\Gamma(\cdot)$ — Eylerning gamma funksiyasi.

Berilgan p(x), f(t), φ(x), q(t) funksiyalar va $\alpha \in (0, 1)$, $\beta \in [0, 1]$ sonlar uchun (1)-(3) boshlang'ich-chegaraviy masalasining yechimini topish masalasi o'rganiladi, bu masala to'g'ri masala deb ataladi.

1-teskari masala. Berilgan p(x), f(t), $\varphi(x)$ funksiyalar va $\alpha \in (0, 1)$, $\beta \in [0, 1]$ sonlar uchun (1)-(3) to'g'ri masalaning yechimida qo'shimcha shartdan:

$$\int_0^l \omega(x)u(x,t)dx = h(t), \qquad t \in [0.T] \qquad (4)$$

foydalanib q(t), $t \in [0, T]$ aniqlash talab etiladi.

Keyingi teskari masalada, q(t) = 0 deb qaraladi.

2-teskari masala. Berilgan p(x), f(t), φ(x) funksiyalar va $\alpha \in (0, 1)$, $\beta \in [0, 1]$ sonlar uchun (1)-(3) to'g'ri masalaning yechimida qo'shimcha shartdan:

$$u(x,T) = g(x), \quad x \in [0, l] \qquad (5)$$

foydalanib p(x), $x \in [0, l]$ aniqlash talab etiladi.

Faraz qilaylik, berilgan φ, f, h, ω va g funksiyalar oldindan berilgan, yetarlicha silliq va qaralayotgan ish davomida quyidagi shartlarni qanoatlantirsin:

A1)$\varphi \in C^3\,[0, l]$, $\varphi^{(4)}(x) \in L^2[0, l]$, $\varphi(0) = \varphi(l)$, $\varphi''(0) = \varphi''(l)$;

A2)p $\in C^3\,[0, l]$, $p^{(4)}(x) \in L^2[\,0, l]$, p(0) = p(l), p''(0) = p''(l);

A3) ω (x) ∈ C^2 [0, l] va $\omega(0) = \omega(l) = 0$;

A4) $(D_{0+}^{\alpha,\beta} h)$ (t) ∈ C[0, T] va $|h(t)| \geq h_0 > 0$, h_0 − berilgan son,

$\int_0^l \omega(x)\varphi(x)dx = (I_{0+,t}^{(1-\beta)(1-\alpha)} h(t)|_{t=0}$;

A5)g ∈ C^2 [0, l], g(0) = g(l) = 0;

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