UMUMLASHGAN RIMANN-LIUVILL KASR HOSILALI DIFFUZIYA TO‘LQIN TENGLAMASI UCHUN TESKARI MASALA

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Klassik kasr hosilalaridan tashqari, turli vaziyatlarda modellarning haqiqatga yaxshiroq mos kelishi uchun bir nechta umumlashmalar kiritildi. Teskari masalalarda, kasr tartibli differensial tenglamalar uchun ko’pincha fazoviy o’zgaruvchilarga yoki vaqtga bog’liq bo’lgan manbani aniqlash masalala ri o’rganiladi. [1]-[6] maqolalarida kasr diffuziya-to’lqin tenglamasi uchun kichik hadi oldidagi koeffitsiyentni aniqlash bo’yicha teskari masalalar o’rganilgan. Ushbu tadqiqotning asosiy natijalari yechimning mavjudligi va yagonaligi teoremalari, shuningdek, yechimning turg’unlik bahosidir.

$\Omega = \{(x,t) : 0 < x < l, 0 < t \le T\}$ sohada quyidagi kasr-differensial tenglamani qaraymiz.

$$(D_{0+}^{\alpha,\beta}u)(x,t) - u_{xx} + q(t)u(x,t) = p(x)f(t), \ (0 < \alpha < 1;\ 0 \le \beta \le 1) \tag{1}$$

boshlang’ich shart:

$$I_{0+,t}^{(1-\alpha)(1-\beta)}u(x,t)|_{t=0} = \varphi(x),\ x \in [0, l] \tag{2}$$

chegaraviy shartlar:

$$u(0,t) = 0,\ u(l,t) = 0,\ 0 \le t \le T, \tag{3}$$

$0 < \alpha < 1$ tartibli va $0 \le \beta \le 1$ tipli umumlashgan Riman-Liuvill kasr differensial operatori(Hilfer) $D_{0+}^{\alpha,\beta}$ quyidagicha aniqlangan:

$$D_{0+}^{\alpha,\beta}u(\cdot,t) = \left(I_{0+,t}^{\beta(1-\alpha)}\frac{\partial}{\partial t}\left(I_{0+,t}^{(1-\beta)(1-\alpha)}u\right)\right)(\cdot,t),$$

$$I_{0+,t}^{\gamma}u(x,t) = \frac{1}{\Gamma(\gamma)}\int_0^t \frac{u(x,\tau)}{(t-\tau)^{1-\gamma}}\,d\tau,\ \gamma\epsilon(0,1).$$

$I_{0+,t}^{\gamma}$ — $u(x,t)$ funksiyadan t bo’yicha olingan Riman-Liuvill kasr integral, $\Gamma(\cdot)$ — Eylerning gamma funksiyasi.

Berilgan $p(x), f(t), \varphi(x)$funksiyalar va $\alpha \in (0,1), \beta \in [0,1]$ sonlar uchun (1)-(3) to’g’ri masalaning yechimida qo’shimcha shartdan:

$$\int_0^l w(x)u(x,t)\,dx = h(t),\quad t \in [0,T], \tag{4}$$

foydalanib q(t), t ∈ [0, T] aniqlash masalasiga teskari masala deyiladi.

Faraz qilaylik, berilgan φ, f, h, w va g funksiyalar oldindan berilgan, yetarlicha silliq va qaralayotgan ish davomida quyidagi shartlarni qanoatlantirsin:

A1) $\varphi(x) \in C^2[0,l]$, $\varphi^{(3)}(x) \in L^2[0,l]$, $\varphi(0) = \varphi(l) = 0$, $\varphi''(0) = \varphi''(l) = 0$;

A2) p(x) ∈ C^2 [0, l], $p^{(3)}$(x) ∈L^2[0, l], p(0) = p(l)=0, p ″(0) = p ″(l)=0;

A3) $w(x) \in C^2$ [0, l] va $w(0) = w(l) = 0$;

A4) $\left(D_{0+}^{\alpha,\beta} h\right)$ (t) ∈ $C[0,T]$ va |h(t)|≥ h_0>0, $h_0 - berilgan\ son,$

$$\int_0^l w(x)\varphi(x)dx = (I_{0+,t}^{(1-\alpha)(1-\beta)} h)(t)|_{t=0};$$

A5) $g \in C^2$ [0, l], $g(0) = g(l) = 0$;

Uzluksiz va silliq funksiyalar uchun vaznli fazolarni ko'rib chiqamiz:

C_γ[a, b] := { f : (a, b) → R: (x − a) $^\gamma$ f(x) ∈ C[a, b], 0 ≤ γ < 1 } ;

$$C^{2,\alpha}(\Omega) = \{u(x,t) : u(\cdot,t) \in C^2(0,l); t \in [0,T] \text{ va } \partial_{0t}^{\alpha} u(x,\cdot) \in C(0,T];\ x \in [0,l]\};$$

$$C^{2,\alpha,\beta}(\Omega) = \{u(x,t): u(\cdot,t) \in C^2\Big((0,l); t \in [0,T] va\ D_{0+}^{\alpha,\beta} u(x,\cdot) \in C_\gamma(0,T];\ x \in [0,l],\ 0 < \alpha \le 1, 0 \le \beta \le 1\Big\}.;$$

$$C_\gamma^0\gamma\ [a, b] = C_\gamma\ [a, b],$$

va bu fazolarda norma quyidagicha aniqlanadi:

$\| f \|_{C_\gamma} = \|$ (x − a) $^\gamma$f(x) $\|_C$, $\| f \|_{C_\gamma^n} = \sum_{k=0}^{n-1} \| f^{(k)} \|C + \| f^{(n)} \|_{C_\gamma}$.

Teorema. A1)-A5) shartlar bajarilsin. U holda T^*∈ (0, T) son mavjud bo'lib, (1)-(4) teskari masalaning yagona yechimi q(t) ∈ C[0, T^*] mavjud.

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