MUSBAT ANIQLANGAN MATRIRSALARNING SHUR TO‘LDIRUVCHISI

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Annotatsiya. Ushbu maqolada Musbat aniqlangan Blok matritsalarni Shur to’ldiruvchisi haqida tushuncha , Musbat aniqlangan matritsasini qo’llash , Musbat aniqlangan matritsasiga oid bir nechta teoremalar va misollar berildi.

Annotation. Annotation. This paper provides an overview of the **Positive Block** matrix, the application of the **Positive** matrix, and several theorems and examples related to the **Positive** matrix.

Kalit so’zlar: Blok matritsa , argument, kompleks, matritsa, transponerlash, musbat matritsa, funksiya, simetrik

Key words: **Block matrix**, argument, complex, matrix, transponder, positive matrix, function, symmetric

M_n da A berilgan bo‘lsa, S_A M_n da anilanib, chiziqli xarita bo‘lsin

$$S_A(X) = A°X, \quad X \in M_n, \tag{1}$$

Bu erda $A°X$ A va X ning Shur ko‘paytmasi. Chiziqli operator normasi haqidagi ta‘rifiga ko‘ra,

$$\|S_A\| = \sup_{\|X\|=1} \| S_A(X)\| = \sup_{\|X\|\le 1} \| S_A(X)\| \tag{2}$$

$$\|A \cdot B\| \le \|A \otimes B\| = \|A\|\|B\| \tag{3}$$

Bizda quyudagi munosabat bor

$$\|S_A\| \le \|A\| \tag{4}$$

$\|S_A\|$ isning aniq qiymatini topish umuman qiyin muammo.

Ba‘zi maxsus holatlar osonroq.

3-teorema (Shurteoremasi). Agar A musbat bo‘lsa, u holda

$$\|S_A\| = max a_{ii} \tag{5}$$

Bo‘ladi.

Isbot. $\|X\| \le 1$ berilgan bo‘lsin. 2.2.1-teoremaga ko’ra $\begin{bmatrix} I & X \\ X^* & I \end{bmatrix} \ge 0$. U holda 1-lemmadan $\begin{bmatrix} A & A \\ A & A \end{bmatrix} \ge 0$ tengsizlik o’rinli bo’lishi kelib chiqadi. Demak, bu ikki blok matritsaning Shur ko‘paytymasi musbat bo‘ladi; ya‘ni,

$$\begin{bmatrix} A \cdot I & A \cdot X \\ (A \cdot X)^* & A \cdot I \end{bmatrix} \ge 0$$

Shunday qilib, 2-taklif bo‘yicha, $\|A \cdot X\| \le \|A \cdot I\| = max a_{ii}$. Ya’ni, $\|S_A\| = max a_{ii}$

4-teorema. A ixtiyoriy matritsa bo‘lsin. U holda ushbu

$$\|S_A\| \leq inf\{\|X\|_c\|Y\|_c : A = X^*Y\} \tag{8}$$

Tengsizlik o‘rinli

Isbot. $A = X^*Y$ berilgan bo‘lsin. U holda

$$\begin{bmatrix} X^*X & A \\ A^* & Y^*Y \end{bmatrix} = \begin{bmatrix} X^* & O \\ Y^* & O \end{bmatrix} \begin{bmatrix} X & Y \\ O & O \end{bmatrix} \geq 0 \tag{9}$$

Endi agar Z $\|Z\| \leq 1$ bo‘lgan har qanday matritsa bo‘lsa, u holda $\begin{bmatrix} I & Z \\ Z^* & I \end{bmatrix} \geq 0$. Shunday qilib, Shur ko‘paytmasi uning 8) musbat matritsi bilan musbat bo‘ladi; ya‘ni,

$$\begin{bmatrix} (X^*X) \cdot I & A \cdot Z \\ (A \cdot Z)^* & (Y^*Y) \cdot I \end{bmatrix} \geq 0$$

Demak, 3 teoremadan

$$\|A \cdot Z\| \leq \|X^*X) \cdot I\|^{\frac{1}{2}} \|(Y^*Y) \cdot I\|^{\frac{1}{2}} = \|X\|_c\|Y\|_c \tag{10}$$

Bundan $\|S_A\| \leq \|X\|_c\|Y\|_c$.

Monotonlik va qavarilik. $\mathcal{L}_{s.a.}(\mathcal{H})$ $\mathcal{H}$ dagi o‘z-o‘ziga qo‘shma (Ermitian) operatorlar fazosi bo‘lsin.

Bu haqiqiy vektor maydoni. Musbat operatorlarning $\mathcal{L}_+(\mathcal{H})$ to‘plami bu fazoda qavariq konus. Qa‘tiy musbat operatorlar to‘plami $\mathcal{L}_{++}(\mathcal{H})$ bilan belgilanadi. Ochiq $\mathcal{L}_{s.a.}(\mathcal{H})$ to‘plam qavariq konus bo‘ladi.

FOYDALANILGAN ADABIYOTLAR

1. Sh.A.Ayupov, B.A.Omirov, A.X.Xudoyberdiyev Chiziqli algebra . Darslik 2023 y.
2. А. С. Мищенко, Эрмитова К-теория. Теория характеристических классов, методы функционального анализа, УМН, 1976, том 31
3. С. П. Новиков , Алгебраическое построение и свойства эрмитовых аналогов Z-теории над кольцами с инволюцией с точки зрения гамильтонового формализма. Некоторые применения к дифференциальной топологии и теории характеристических классов. I, Изв. АН, сер. матем.
4. Ф.Гантмахер. Теория матриц. ЛитРес. 2018, 581 с.
5. **Leonard E. Fuller. Basic Matrix Theory ЛитРес. 2017, 256 с.**
6. **R. Bhatia Matrix Analysis Springer-Verlag, New York, 1997**
7. Richard Bellman. Introduction to Matrix Analysis in 1997 426 p.
8. Sh.A.Ayupov, B.A.Omirov, A.X.Xudoyberiyev, F.H.Haydarov Algebra va sonlar nazariyasi. Toshkent 2019 296 s.
9. Yisong Yang. A Matrix Trace Inequality, Journal of Mathematical Analysis and Applications, 133 (1988), pp. 573-574.

10. N.Tosheva, M.Shodiyev. Ermit matritsalari va ularning xossalarini “bumerang” metodi orqali oʻrganish PEDAGOGIK MAHORAT Ilmiy-nazariy va metodik jurnal MAXSUS SON
11. Ando T. Concavity of certain maps on positive definite matrices and applications to Hadamard products. Linear Algebra and Applications. Vol. 26, 1976, P. 203-241.