MATRITSANING FUNKSIYASI

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Annotatsiya. Ushbu maqolada Musbat aniqlangan Blok matritsalarni Shur to'ldiruvchisi haqida tushuncha , Musbat aniqlangan matritsasini qo'llash , Musbat aniqlangan matritsasiga oid bir nechta teoremalar va misollar berildi.

Annotation. Annotation. This paper provides an overview of the **Positive Block** matrix, the application of the **Positive** matrix, and several theorems and examples related to the **Positive** matrix.

Kalit so'zlar: Blok matritsa , argument, kompleks, matritsa, transponerlash, musbat matritsa, funksiya, simetrik

Key words: **Block matrix**, argument, complex, matrix, transponder, positive matrix, function, symmetric

Simmetrik matritsalarning funktsiyalari. Har bir haqiqiy simmetrik matritsani quyudagicha ifodalanishi mumkin

$$A = T\begin{bmatrix} \lambda_1 & 0 & \ddots & 0 \\ 0 & \lambda_2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix}T^T \qquad (1)$$

Bu yerda T - ortogonal matritsa. Bundan quyudagi xulosa kelib chiqadi

$$A^k = T\begin{bmatrix} {\lambda_1}^k & 0 & \ddots & 0 \\ 0 & {\lambda_2}^k & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & {\lambda_n}^k \end{bmatrix}T^T \qquad (2)$$

har qanday k o'zgarmas son uchun . Demak, har qanday $f(z)$ analitik funktsiya uchun $f(A)$ni quyudagicha aniqlashimiz mumkin

$$f(A) = T\begin{bmatrix} f(\lambda_1) & 0 & \ddots & 0 \\ 0 & f(\lambda_2) & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & f(\lambda_n) \end{bmatrix}T^T \qquad (3)$$

Bunda $f(\lambda_n)$ skalyar funksiyalardek aniqlangan.

Teskari matritsa. Bunda simmetrik matritsa kabi aniqlanadi. Teskari matritsani quyidagicha belgilashga olib kelinadi.

$$A^{-1} = T\begin{bmatrix} \lambda_1{}^{-1} & 0 & \ddots & 0 \\ 0 & \lambda_2{}^{-1} & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \lambda_n{}^{-1} \end{bmatrix} T^T \qquad (4)$$

Bunda barcha $\lambda_i \;\; i = \overline{1,n}$ lar noldan farqli sonlar.

Ko‘rinib turibdiki, A^{-1} teskarisining tegishli xususiyatlariga ega, ya‘ni

$$AA^{-1} = I$$

2-tenglikga ko‘ra $A^{\frac{1}{2}}$ ifoda quyudagicha aniqlangan

$$A^{\frac{1}{2}} = T\begin{bmatrix} \lambda_1{}^{\frac{1}{2}} & 0 & \ddots & 0 \\ 0 & \lambda_2{}^{\frac{1}{2}} & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \lambda_n{}^{\frac{1}{2}} \end{bmatrix} T^T \qquad (5)$$

shu tarzda $B^2 = A$ munosabatni qanoatlantiruvchi matritsani olish, ya‘ni B musbat aniqlangan matritsa bo‘ladi, agarda A musbat aniqlangan bo‘lsa.

O‘ziga xoslik masalasi hali ham mavjud. Buni hal qilish uchun biz quyidagicha davom etira olamiz . B simmetrik bo‘lgani uchun uni quyudagicha yoza olamiz:

$$B = S\begin{bmatrix} \mu_1 & 0 & \ddots & 0 \\ 0 & \mu_2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \mu_n \end{bmatrix} S^T \qquad (6)$$

Bunda S orthogonal. Bundan esa quyudagi xulosa kelib chiqadiki, B va $B^2 = A$ umumiy. Demak, ikkalasini ham bir xil ortogonal matritsa T orqali diagonal shaklga keltirilishi mumkin.

1-teorema Agar $A = (a_{ij}), \;\; B = (b_{ij})$ matritsalar musbat aniqlangan bo‘lsa, u holda $A = (a_{ij}b_{ij})$ musbat aniqlangan bo‘ladi.

Isbot. Quyudagi tenglikdan foydalanamiz

$$\sum_{i,j=1}^{n} a_{ij}b_{ij}x_ix_j = \sum_{i,j=1}^{n} b_{ij}x_ix_j\left(\sum_{k=1}^{n}\lambda_k t_{ik}t_{jk}\right) = \sum_{k=1}^{n}\lambda_k\left(\sum_{i,j=1}^{n} b_{ij}x_it_{ik}x_j\,t_{jk}\right)$$

$$\lambda_k > 0 \qquad (7)$$

x_it_{ik} o‘zgaruvchilarning kvadratik formasi sifatida $\sum_{i,j=1}^{n} b_{ij}x_it_{ik}x_j$ ifoda musbat aniqlangan bo‘ladi agarda x_it_{ik} miqdor nolga teng bo‘lmasa. Bundan

$$\sum_{i,k} x_i{}^2\, t_{ik}{}^2 = \sum_i x_i{}^2 \sum_k t_{ik}{}^2 = \sum_i x_i{}^2 \qquad (8)$$

Bundan biz barcha x_i nolga teng bo‘lmasa, u holda barcha i uchun to‘g‘ri bo‘lishi mumkin emasligini ko‘ramiz.

Bundan esa musbat aniqlanganligi kelib chiqadi.

Asosiy skalyar funksiyalar. A xarakteristik ko‘phadi orqali aniqlangan skalyar funksiyalarining xossalari ko‘ramiz

$$|A-\lambda I| = \lambda^n - \varphi_1(A)\lambda^{n-1} + \varphi_2(A)\lambda^{n-2} + \cdots + (-1)^n\varphi_n(A) \qquad (9)$$

Bu turdagi matritsalar kvadrat matritsadan tashqari yana simmetrik bo‘lishi kerak.
Bu ko‘phadning ildizlari va koeffitsentlkari orasida quyudagi bog‘lanish kelib chiqadi

$$\varphi_1(A) = \lambda_1 + \lambda_2 + \cdots + \lambda_n$$

$$\varphi_2(A) = \sum_{i\neq j} \lambda_i\lambda_j$$

$$\vdots$$

$$\varphi_n(A) = \lambda_1 \cdot \lambda_2 \cdot \ldots \cdot \lambda_n$$

Boshqa tomondan, ko‘pxad xarakteristikasini λ^{n-1} yozilgan termini, $|A-\lambda I|$ determinantining kengayishidan kelib chiqadi .
Bundan esa quyudagi kelib chiqadi.

$$\varphi_1(A) = a_{11} + a_{22} + \cdots + a_{nn} \qquad (10)$$

$\lambda = 0$ bo‘lgan hol uchun

$$\varphi_n(A) = |A|$$

Chiziqli funksiya $\varphi_1(A)$ matritsalar nazariyasida katta ahamiyatga ega
Bu funksiya A matrirsaning izi deb nomlanadi va trA orqali belgilanadi.

2-teorema. Agar A va B ikkita nxn musbat aniqlangan matritsa bo‘lsa, u holda

(I) $tr(AB) > 0\ va$

(II) $\sqrt{tr(AB)} < \frac{trA+trB}{2}$

bo‘ladi.
Isbot. P shunday ortogonal matritsa bo‘lsin

$$P'AP = diag(k_1, \ldots, k_n) = J$$

Keyin

$$tr(\,AB) = tr(P'ABP) = tr(\,P'APP'BP) = tr(JC), \quad (i)$$

bunda $C = P'BP$ ham musbat aniqlangan matritsadir. Endi bizda

$$tr(JC) = diag(k_1, \ldots, k_n)\cdot(c_{ij}) = \begin{pmatrix} k_1c_{11} & \cdots & * \\ \vdots & \ddots & \vdots \\ * & \cdots & k_nc_{nn} \end{pmatrix}$$

Bu bilan birga

$$(JC) = k_1c_{11} + \cdots + k_nc_{nn} > 0 \qquad (ii)$$

Boshqa tomondan quyudagi o‘rinli

$$(trJ + trC)^2 - 4tr(JC) = (k_1 - c_{11})^2 + \cdots + (k_n - c_{nn})^2\ (iii)\ musbat\ hadlar$$

(I) Dan (i) va (ii) berilgan. (i) va (iii) ni qo‘llab quyudagini yozamiz

$$tr(AB) = tr(JC) < \frac{(trJ + trC)^2}{2} = \left(\frac{trA + trB}{2}\right)^2$$

Teorema isbotlandi.

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