MUSBAT ANIQLANGAN MATRITSALAR ORQALI QURULGAN BA'ZI TENGSIZLIKLAR

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Annotatsiya. Ushbu maqolada Musbat aniqlangan Blok matritsalarni Shur to'ldiruvchisi haqida tushuncha , Musbat aniqlangan matritsasini qo'llash , Musbat aniqlangan matritsasiga oid bir nechta teoremalar va misollar berildi.

Annotation. Annotation. This paper provides an overview of the **Positive Block** matrix, the application of the **Positive** matrix, and several theorems and examples related to the **Positive** matrix.

Kalit so'zlar: Blok matritsa , argument, kompleks, matritsa, transponerlash, musbat matritsa, funksiya, simetrik

Key words: **Block matrix**, argument, complex, matrix, transponder, positive matrix, function, symmetric

A va B ixtiyoriy ikkita matritsa bo'lsin, AB va BA bir xil xos qiymatlarga ega bo'lsin. Demak, $f(A)$ matritsalar fazosida faqat A ning xos qiymatlariga bog'liq bo'lgan har qanday funksiya bo'lsa, $f(AB) = f(BA)$ bo'ladi. Bunday funksiyalarga spektral radius, iz va aniqlovchi misol bo'la oladi. Agar A normal bo'lsa, u holda spektral radius $spr(A)$ $||A||$ ga teng. Bundan foydalanib, quyidagi ikkita teoremani isbotlashimiz mumkin.

1-teorema A va B ixtiyoriy ikkita matritsa va AB ko'paytma normal bo'lsin. Keyin, har bir unitar invariant norma uchun quyudagi tengsizlik

$$|||AB||| \leq |||BA||| \qquad (1)$$

O'rinli bo'ladi

Isbot. Operator normasi uchun quyudagi had mavjud

$$\|AB\| = spr(AB) = spr(BA) \leq \|BA\|$$

Umumiy isbot ko'proq ish talab qiladi. AB normal bo'lgani uchun $s_j(AB) = \left|\lambda_j(AB)\right|$, bu erda $|\lambda_1(AB)| \geq \cdots \geq |\lambda_n(AB)|$ AB ning xos qiymatlari kamayish tartibida joylashtirilgan. Lekin $\left|\lambda_j(AB)\right| = \left|\lambda_j(BA)\right|$. Veylning asosiy teoremasi bo'yicha $|\lambda(AB)|$ vektor $s(BA)$ vektori bilan kuchsiz kattalashgan. Demak, bizda $s(AB) \prec_w s(BA)$ kuchsiz ko'rsatkichlar mavjud. Bundan (1) tengsizlik kelib chiqadi. ■

2-teorema. A, B ixtiyoriy ikkita matritsa bo'lsin, AB ko'paytma Ermitian bo'lsin. So'ng, har bir unitar invariant norma uchun bizda quyudagi mavjud

$$|||AB||| \leq |||Re(BA)||| \quad (2)$$

Bo‘ladi

Isbot. BA ning xos qiymatlari AB Ermit matritsasining xos qiymatlari bilan bir xil bo‘lib, barchasi haqiqiydir. Shunday qilib, bizda $\lambda(BA) \prec \lambda(ReBA)$ ko‘payish mavjud. Bundan bizda kuchsiz ko‘paytirish mavjud $\lambda(BA) \prec_w \lambda(ReBA)$. Argumentning qolgan qismi oldingi teoremani isbotlash bilan bir xil. ■

3-teorema. Ixtiyoriy ikkita $A.B$ matritsalari uchun, ushbu

$$\lim_{n \to \infty} \left(exp\frac{A}{m}exp\frac{B}{m}\right)^m = \exp(A+B) \quad (3)$$

Tenglik o‘rinli bo‘ladi.

Isbot. Ixtiyoriy ikkita X, Y matritsalari va $m = 1,2, \dots$ uchun.

$$X^m - Y^m = \sum_{j=0}^{m-1} X^{m-1-j}(X-Y)Y^j$$

Tenglik o‘rinli

Buning yordamida biz quyudagi natijani olamiz

$$\|X^m - Y^m\| \leq mM^{m-1}\|X-Y\| \quad (4)$$

Bu yerda $M = max(\|X\|, \|Y\|)$

Quyudagilar berilgan bo‘lsin $X_m = exp\left(\frac{A+B}{m}\right), Y_m = exp\frac{A}{m}exp\frac{B}{m},\ m = 1,2,3 \dots$ $\|X_m\|\ va\ \|Y_m\|$ ikkalasi ham yuqoridan $exp\left(\frac{\|A\|+\|B\|}{m}\right)$ bilan chegaralangan. Eksponensial funktsiya uchun quvvat seriyasining kengayishidan shuni ko‘ramizki,

$$\begin{aligned} X_m - Y_m &= 1 + \frac{A+B}{m} + \frac{1}{2}\left(\frac{A+B}{m}\right)^2 + \cdots \\ &- \left\{\left[1 + \frac{A}{m} + \frac{1}{2}\left(\frac{A}{m}\right)^2 + \cdots\right]\left[1 + \frac{B}{m} + \frac{1}{2}\left(\frac{B}{m}\right)^2 + \cdots\right]\right\} \\ &= O\left(\frac{1}{m^2}\right)\ m\ dan\ kattalar\ uchun \end{aligned}$$

Demak, (4) tengsizlikdan foydalanib, ushbu tengsizlik

$$X_m^m - Y_m^m \leq m\, exp(\|A\| + \|B\|)O\left(\frac{1}{m^2}\right)$$

O‘rinli bo‘lishini ko‘ramiz.

Bu $m \to \infty$ da nolga teng bo‘ladi. Lekin barcha $m$lar uchun $X_m^m = exp(A+B)$. Demak, $\lim_{m\to\infty} Y_m^m = exp(A+B)$ Bu esa teoremani isbotlaydi. ■

Musbat matritsalar ko‘paytmasi. Ushbu bo‘limda biz ikkita musbat matritsalar ko‘paytmasining normasi, spektral radiusi va xos qiymatlari uchun ba‘zi tengsizliklarni isbotlaymiz.

4-teorema A, B musbat matritsalar berilgan bo‘lsin. U holda ushbu

$$\|A^sB^s\| \leq \|AB\|^s \qquad bunda \qquad 0 \leq s \leq 1 \quad (5)$$

Tengsizlik o‘rinli bo‘ladi.

Isbot.

$$D = \{s\colon 0 \le s \le 1, \qquad \|A^sB^s\| \le \|AB\|^s\}$$

U holda D [0,1] qism to‘plamda yopiq va 0 va 1 nuqtalarni o‘z ichiga oladi. Demak, teoremani isbotlash uchun, agar s va t D da bo‘lsa, $\frac{s+t}{2}$ bo‘lishini isbotlash kifoya. Bizda ushbu tengsizlik o‘rinli

$$\left\|A^{\frac{s+t}{2}}B^{\frac{s+t}{2}}\right\|^2 = \left\|B^{\frac{s+t}{2}}A^{s+t}B^{\frac{s+t}{2}}\right\| = spr\left(B^{\frac{s+t}{2}}A^{s+t}B^{\frac{s+t}{2}}\right) = spr(B^sA^{s+t}B^t)$$
$$\le \|B^sA^{s+t}B^t\| \le \|B^sA^s\|\|A^tB^t\| = \|A^sB^s\|\|A^tB^t\|$$

Birinchi bosqichda, $\|T\|^2 = \|T^*T\|$ oxirgi bosqichda barcha Tlar uchunesa $\|T^*\| = \|T\|$ munosabatidan foydalandik. Agar s, t D da bo‘lsa, bu shuni ko‘rsatadi va bu teoremani isbotlaydi. ■

5-teorema A, B musbat matritsalar berilgan bo‘lsin. U holda

$$\|A^sB^s\| \le \|AB\|^s \qquad bunda \qquad 0 \le s \le 1 \qquad (6)$$

Isbot.

$$D = \{s\colon 0 \le s \le 1, \qquad \|A^sB^s\| \le \|AB\|^s\}$$

U holda D [0,1] qism to‘plamda yopiqva 0 va 1 nuqtalarni o‘z ichiga oladi. Demak, teoremani isbotlash uchun, agar s va t D da bo‘lsa, $\frac{s+t}{2}$ bo‘lishini isbotlash kifoya. Bizda quyudagi mavjud:

$$\left\|A^{\frac{s+t}{2}}B^{\frac{s+t}{2}}\right\|^2 = \left\|B^{\frac{s+t}{2}}A^{s+t}B^{\frac{s+t}{2}}\right\| = spr\left(B^{\frac{s+t}{2}}A^{s+t}B^{\frac{s+t}{2}}\right) = spr(B^sA^{s+t}B^t)$$
$$\le \|B^sA^{s+t}B^t\| \le \|B^sA^s\|\|A^tB^t\| = \|A^sB^s\|\|A^tB^t\|$$

Birinchi bosqichda, $\|T\|^2 = \|T^*T\|$ oxirgi bosqichda barcha Tlar uchun esa $\|T^*\| = \|T\|$ munosabatidan foydalandik. Agar s, t D da bo‘lsa, bundan esa teorema isbotlanadi. ■

1-misol. $A = \begin{pmatrix} 12 & 6 & 8 \\ 9 & 7 & 11 \\ 13 & 7 & 13 \end{pmatrix}$ va $B = \begin{pmatrix} 7 & 5 & 6 \\ 8 & 7 & 9 \\ 9 & 6 & 8 \end{pmatrix}$ matritsalar va $s = 0.5$ berilgan bo‘lsin u holda 5-teorema shartlari bajarilishini tekshiring.

$$A = A^{\frac{1}{2}} \cdot A^{\frac{1}{2}} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 3 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 12 & 6 & 8 \\ 9 & 7 & 11 \\ 13 & 7 & 13 \end{pmatrix}$$

$$B = B^{\frac{1}{2}} \cdot B^{\frac{1}{2}} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 5 & 6 \\ 8 & 7 & 9 \\ 9 & 6 & 8 \end{pmatrix}$$

Bulardan esa $A^{\frac{1}{2}} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 3 \end{pmatrix}$ va $B^{\frac{1}{2}} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 2 \end{pmatrix}$ ekanligi kelib chiqadi.

$$\left\|A^{\frac{1}{2}}B^{\frac{1}{2}}\right\| \le \|AB\|^{\frac{1}{2}}$$

Shartni tekshiramiz.

$$A^{\frac{1}{2}} \cdot B^{\frac{1}{2}} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 9 & 6 & 7 \\ 8 & 7 & 9 \\ 11 & 7 & 10 \end{pmatrix}$$

$$\left\| A^{\frac{1}{2}} \cdot B^{\frac{1}{2}} \right\| = \left| A^{\frac{1}{2}} \cdot B^{\frac{1}{2}} \right| = \begin{vmatrix} 9 & 6 & 7 \\ 8 & 7 & 9 \\ 11 & 7 & 10 \end{vmatrix} = 15$$

$$A \cdot B = \begin{pmatrix} 12 & 6 & 8 \\ 9 & 7 & 11 \\ 13 & 7 & 13 \end{pmatrix} \cdot \begin{pmatrix} 7 & 5 & 6 \\ 8 & 7 & 9 \\ 9 & 6 & 8 \end{pmatrix} = \begin{pmatrix} 204 & 150 & 190 \\ 218 & 160 & 205 \\ 264 & 192 & 245 \end{pmatrix}$$

$$\|A \cdot B\| = |A \cdot B| = \begin{vmatrix} 204 & 150 & 190 \\ 218 & 160 & 205 \\ 264 & 192 & 245 \end{vmatrix} = 900 \qquad \|AB\|^{\frac{1}{2}} = \sqrt{900} = 30$$

Haqiqatdan ham $15 < 30$ teorema bajariladi.

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