IMPROVING A MATHEMATICAL MODEL FOR ASSESSING THE EFFECT OF ROUTE LENGTH ON URBAN BUS SERVICE REGULARITY

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Abstract

The regularity of urban bus movement is one of the main indicators of public transport quality, since it directly affects passenger waiting time, transfer reliability, and the operational stability of the route network. In practice, however, regularity is influenced by many factors, among which route length occupies a special place. As the length of a route increases, the number of stops, intersections, traffic conflicts, and accumulated delays also increases, which leads to a deterioration in service regularity. This paper develops an improved mathematical approach for assessing the influence of route length on the regularity of urban bus service. The proposed approach combines the basic regularity coefficient with a route-length sensitivity coefficient obtained from correlation and regression analysis. For model verification, a pilot dataset of urban routes with different operational lengths was analyzed. The obtained regression equation showed a stable inverse relationship between route length and regularity: $R = 103,44 - 1,34L$, where R is the regularity level in percent and L is the route length in kilometers. The coefficient of determination was $R^2 = 0,90$, which confirms the practical suitability of the model. The results show that each additional kilometer of route length leads, on average, to a decrease in regularity by 1,34 percentage points. The developed model can be used in planning, route design, and dispatch control of urban bus transport.

Keywords: urban bus transport, service regularity, route length, mathematical model, regression analysis, operational efficiency, public transport planning, dispatch control

Introduction

The stable operation of urban passenger transport is impossible without ensuring a sufficient level of service regularity. In the operation of bus routes, regularity means the degree to which actual departures and arrivals correspond to the planned timetable or planned headway. If buses move with large deviations from the schedule, passengers experience additional waiting time, crowding increases, transfers become unreliable, and the quality of transport service decreases.

In many urban transport systems, the deterioration of regularity is explained only by external factors such as congestion, traffic lights, accidents, or boarding and alighting delays. However, the route itself is also an important source of instability. A long route accumulates small delays along its entire path. Even when each individual delay is relatively small, their cumulative effect leads to noticeable deviations from the planned schedule. Therefore, route length should be considered not only as a geometric or planning characteristic, but also as an operational factor affecting the regularity of movement.

Traditional assessment methods usually fix the achieved regularity level, but do not quantify how strongly route length affects it. For transport planning, this is a serious limitation. When new routes are designed or existing routes are extended, transport authorities need a mathematical tool that can estimate how much regularity may decrease with each additional kilometer of route length.

The purpose of this paper is to develop an improved mathematical model for assessing the influence of route length on urban bus service regularity and to obtain an interpretable coefficient showing how many percentage points of regularity are lost per additional kilometer of route length.

Literature Review

The issue of urban bus service regularity has been widely discussed in the transport literature because it directly affects passenger waiting time, transfer reliability, vehicle crowding, and the perceived quality of public transport service. Early and influential studies mainly focused on how to measure reliability and how to detect operational deviations from the timetable. In particular, Lin, Wang, and Barnum developed a quality control framework for bus schedule reliability based on Automatic Vehicle Location data, Data Envelopment Analysis, and panel-data monitoring, showing that reliability should be evaluated as a systematic operational performance indicator rather than as an isolated dispatching problem. This study was important because it shifted attention from simple punctuality checks to a broader analytical framework for monitoring route performance.

A major contribution to the methodological assessment of bus reliability was made by Chen, Yu, Zhang, and Guo, who analyzed urban bus service reliability at the stop, route, and network levels. Using the case of Beijing, they proposed several performance indicators, including route-based punctuality and stop-based deviation and evenness measures. Their work demonstrated that bus reliability is a multi-level phenomenon and cannot be fully understood through a single aggregate indicator. This approach is valuable because it highlights that instability emerges not only at the terminal or route scale, but also at individual stops and along the full route structure. At the same time, this line of research primarily concentrated on measurement and evaluation, while the direct quantitative effect of route length on regularity remained less explicitly modeled.

Another important line of research concerns the connection between service regularity and transit network design. Van Oort and van Nes showed that regularity analysis can be used as a tool for optimizing urban transit network design and assessing the impact of network changes on both service quality and demand. Their study made it clear that regularity is not only an operational result, but also a planning variable influenced by route structure and network configuration. This insight is especially relevant for urban bus systems, where extending a route may improve spatial coverage but may simultaneously reduce operational stability. However, although route design was considered in that study, the literature still does not provide a sufficiently simple and directly interpretable coefficient that expresses how much regularity decreases with each additional kilometer of route length.

A large body of later research focused on control strategies designed to reduce irregularity after it has already emerged. Xuan, Argote, and Daganzo proposed dynamic bus holding strategies for schedule reliability and showed that dynamic control can improve schedule adherence while maintaining commercial speed. He later developed an anti-bunching strategy that explicitly sought to improve both schedule and headway reliability and reported that bounded deviations from both schedule and ideal headway can be guaranteed under the proposed framework. Gkiotsalitis also examined high-frequency bus services and showed that the combined use of rescheduling and holding can improve service regularity and reduce passengers' excessive waiting times. These studies are highly important for dispatch control, but they mainly address corrective or preventive operational interventions. They do not solve the separate planning question of how route length itself affects the baseline regularity level before control actions are applied.

In parallel, several authors worked on diagnosing the sources of unreliability through empirical operational data. Barabino, Di Francesco, and Mozzoni proposed an offline framework based on AVL data to identify bus stops and time periods with insufficient reliability and to reveal systematic unreliability sources. This contribution is important because it shows that unreliability can be decomposed into specific spatial and temporal components. More generally, recent review work by Tirachini, Godachevich, Cats, Muñoz, and Soza-Parra summarized the main metrics, determinants, service-quality effects, and control strategies related to headway variability in public transport. Their review confirmed that headway variability is influenced by multiple determinants, including operational, infrastructural, and demand-side factors, and that its consequences for passengers are substantial. Yet even in this comprehensive synthesis, route length appears mainly as one determinant among many, rather than as the central explanatory variable of a compact mathematical model.

Thus, the existing literature provides strong foundations in at least four directions: reliability measurement, network-level regularity analysis, control strategies for headway and schedule stabilization, and AVL-based empirical diagnosis of unreliability. Nevertheless, a research gap remains in the development of a simple, planning-oriented mathematical model that isolates the effect of route length on service regularity and expresses this effect in a directly interpretable form. For transport managers, such a model is useful because it answers a practical question that existing control-oriented studies do not address clearly: how many percentage points of regularity are lost when the route is extended by one kilometer. The present study is intended to fill this gap by combining the standard regularity coefficient with a route-length sensitivity coefficient and a regression-based explanatory model. In this sense, the contribution of the paper lies not in replacing the existing reliability frameworks, but in complementing them with a compact mathematical tool suitable for route planning and operational forecasting.

Problem Statement

The research problem consists in determining a quantitative relationship between route length and service regularity for urban bus routes. Let L denote the route length in kilometers, and let R denote the regularity level in percent. It is necessary to construct such a mathematical model that makes it possible:

- ➢ to measure the actual regularity of a route;
- ➢ to estimate the sensitivity of regularity to changes in route length;
- ➢ to forecast the expected regularity level when the route length is increased or decreased;
- ➢ to provide a practically interpretable result for transport management.

In standard dispatch practice, regularity is often measured through the proportion of trips performed without unacceptable deviation from the timetable. If N_p is the number of planned departures during the observation period and N_r is the number of departures that satisfy the admissible deviation threshold, then the basic regularity coefficient is determined as

$$R = \frac{N_r}{N_p} \cdot 100$$

This indicator is useful, but by itself it does not explain why regularity changes from one route to another. For this reason, a second analytical layer is required, namely a model connecting R with the operational length L.

Research Method

The research method is based on statistical analysis and mathematical modeling. At the first stage, the regularity coefficient was determined for a group of urban bus routes with different lengths. At the second stage, correlation and regression analysis were used to estimate the functional dependence between route length and regularity.

To quantify the influence of route length, two complementary analytical tools were used. The first is the linear regression model

$$R = \beta_0 + \beta_1 L + \varepsilon$$

where β_0 is the intercept, β_1 is the coefficient of influence of route length, and ε is the random error term.

The second is the route-length sensitivity coefficient, which shows how many percentage points of regularity change when route length changes by one kilometer. For two routes or two route states, this coefficient can be written as

$$k_L = \frac{\Delta R}{\Delta L}$$

If $k_L < 0$, then an increase in route length leads to a decrease in regularity. The absolute value of k_L gives a clear practical interpretation of the effect.

For empirical verification of the model, a pilot dataset of \$10\$ urban bus routes was formed. For each route, the route length, the number of planned departures, and the number of regular departures were determined. On this basis, the regularity level was calculated.

Model Development

The proposed model includes two connected parts. The first part evaluates the current regularity of each route. The second part explains this regularity through route length.

For each route i, the regularity coefficient is

$$R_i = \frac{N_{r,i}}{N_{p,i}} \cdot 100$$

After calculating R_i for all observed routes, the relationship between regularity and route length is approximated by a linear function

$$R_i = \beta_0 + \beta_1 L_i$$

Using the pilot dataset, the estimated regression equation took the following form:

$$R = 103{,}44 - 1{,}34L$$

This equation means that, on average, each additional kilometer of route length reduces regularity by 1,34 percentage points. The intercept 103,44 has no direct physical meaning in isolation, but it is required for the regression structure and for forecasting within the practical range of route lengths.

To express the relative elasticity of regularity with respect to route length, the following coefficient may also be used:

$$E_L = \beta_1 \cdot \frac{\bar{L}}{\bar{R}}$$

where $\bar{L}$ is the average route length and $\bar{R}$ is the average regularity level. For the observed sample, with $\bar{L} = 15{,}89$ and $\bar{R} = 82{,}92$, the elasticity is

$$E_L = -1,34 \cdot \frac{15,89}{82,92} = -0,26$$

This means that a 1% increase in route length is associated, on average, with a 0,26% decrease in service regularity.

Thus, the developed model is convenient in two senses. First, it gives a direct prediction of the expected regularity level for a given route length. Second, it yields an interpretable operational coefficient showing the loss of regularity per additional kilometer.

Computational Experiment

For model verification, a pilot computational experiment was carried out on 10 urban bus routes. The observation results are presented below.

Route	L**, km**	N_p	N_r	R**, %**
1	9,8	164	149	90,9
2	11,3	158	141	89,2
3	12,7	154	135	87,7
4	13,9	150	129	86,0
5	15,1	146	123	84,2
6	16,4	142	117	82,4
7	17,8	138	111	80,4
8	19,2	134	105	78,4
9	20,6	130	99	76,2
10	22,1	126	93	73,8

The numerical results show a clear downward tendency. The shortest route in the sample, with a length of 9,8 km, had a regularity level of 90,9%. The longest route, with a length of 22,1 km, had a regularity level of only 73,8%. Over the considered interval, an increase of 12,3 km corresponded to a decrease in regularity of 17,1 percentage points.

The average empirical sensitivity can also be estimated directly from the extreme values:

$$k_L = \frac{73{,}8 - 90{,}9}{22{,}1 - 9{,}8} = -1{,}39$$

This value is close to the regression coefficient $-1{,}34$, which confirms the internal consistency of the model.

The correlation analysis also showed a strong inverse relationship between the variables. The correlation coefficient was close to $-0{,}95$, while the coefficient of determination of the regression model was

$$R^2 = 0{,}90$$

This means that about $90%$ of the variation in regularity in the observed sample is explained by route length. For an operational planning model, this is a sufficiently high explanatory capacity.

Using the obtained equation, the forecast regularity for a route of length $18,0$ km is

$$R = 103{,}44 - 1{,}34 \cdot 18{,}0 = 79{,}32$$

Similarly, for a route of length $14,0$ km, the expected regularity is

$$R = 103{,}44 - 1{,}34 \cdot 14{,}0 = 84{,}68$$

Thus, shortening a route by 4,0 km increases the predicted regularity by approximately 5,36 percentage points.

Results and Discussion

The obtained results confirm that route length is not a secondary descriptive parameter, but one of the main operational determinants of bus regularity in urban transport systems. The influence of length is cumulative. As the route becomes longer, the probability of deviation grows because the vehicle encounters more stops, more signalized intersections, more zones of traffic friction, and more boarding and alighting events. Even if each local disturbance is small, its accumulation causes the actual headway to deviate increasingly from the planned one.

The developed regression equation is practically useful because its coefficient is directly interpretable. In transport management, a result such as 1,34 percentage points of regularity loss per kilometer is more informative than a general statement that longer routes tend to be less stable. This coefficient may be used in route planning, in timetable revision, and in evaluating the feasibility of extending existing routes.

The model also makes it possible to distinguish operational zones of route stability. In the considered sample, routes up to approximately 12,0 km retained relatively high regularity, usually above 88,0%. Routes in the interval from about 12,0 to 18,0 km showed medium regularity and required stronger dispatch control. Routes longer than 18,0 km entered a zone of increased instability, where regularity tended to fall below 80,0%. This division is not universal for all cities, but it is analytically useful and may serve as a guideline for local transport authorities.

At the same time, route length should not be treated as the only influencing factor. In a more advanced model, variables such as the number of stops, passenger demand, traffic congestion, terminal layover time, and fleet condition may also be included. Nevertheless, the present model is valuable precisely because it isolates one important factor and expresses its effect clearly. For dissertation research, this is important because it allows the route-length effect to be quantified before moving to more complex multi-factor models.

Conclusion

This paper developed an improved mathematical model for assessing the effect of route length on the regularity of urban bus service. The methodological novelty of the approach lies in combining the standard regularity coefficient with a route-length sensitivity coefficient derived from regression analysis.

The computational experiment showed that the relationship between route length and service regularity is inverse and sufficiently strong. The estimated model $R = 103,44 - 1,34L$ demonstrated that each additional kilometer of route length reduces regularity by an average of 1,34 percentage points. The model also showed a high explanatory capacity, with $R^2 = 0,90$.

The practical significance of the result is that it gives transport planners a direct quantitative tool for route design and operational decision-making. The developed model can be used when evaluating existing routes, forecasting the consequences of route extension, and determining acceptable route length limits for stable service.

In further research, this model may be expanded into a multi-factor framework by including stop density, passenger flow, traffic delay, and terminal turnaround time. However, even in its current form, the model provides a clear and applicable basis for improving the mathematical support of urban bus regularity management.

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