IMPROVING AN INTEGRAL COEFFICIENT MODEL FOR ASSESSING URBAN BUS SERVICE REGULARITY

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Abstract

The assessment of urban bus service regularity is usually based on a simple ratio between the number of trips operated within the permissible deviation and the total number of planned trips. Although this indicator is convenient, it does not reflect the full operational picture because it ignores the magnitude of headway deviations, travel time instability, and the presence of missed trips. As a result, routes with different operational quality may receive similar regularity scores. This paper develops an improved integral coefficient model for evaluating the regularity of urban bus routes. The proposed model combines four components: the basic regularity coefficient, the headway stability coefficient, the travel time stability coefficient, and the trip completion coefficient. Each component is normalized and included in a weighted integral index. A computational experiment was carried out for five urban bus routes. The results showed that the traditional regularity coefficient alone overestimates the quality of some routes, whereas the integral coefficient gives a more balanced and realistic assessment. For example, one of the studied routes had a basic regularity level of $89,0%$, but its integral regularity score was only $84,4%$ because of significant headway and travel time instability. The proposed model can be used in route monitoring, comparative performance analysis, dispatch diagnostics, and transport planning.

Keywords: urban bus transport, service regularity, integral coefficient, mathematical model, headway stability, travel time stability, trip completion, operational assessment

Introduction

The quality of urban passenger transport depends not only on the number of buses on a route and the planned timetable, but also on the regularity with which vehicles actually move along the route. In practical operation, service regularity is one of the main performance indicators because it affects passenger waiting time, transfer coordination, crowding, and the perceived reliability of the entire transport system.

In many transport agencies, regularity is evaluated by a simple percentage indicator showing how many departures were performed within an admissible deviation from the timetable or from the target headway. This indicator is useful as a primary operational measure, but it has a serious limitation. It reduces a complex transport process to only one characteristic and ignores several important aspects of route performance. For example, two routes may have similar percentages of regular departures, while one of them suffers from large headway fluctuations, unstable travel times, or a higher number of missed trips. In such a case, the same percentage value does not reflect the real difference in operational quality.

For this reason, the problem of improving the regularity assessment model becomes important. The assessment method should not only record whether the trip was regular or irregular, but should also take into account how stable the movement was, how strongly the actual headway deviated from the planned interval, how predictable the travel time was, and whether all planned trips were actually performed.

The purpose of this paper is to develop an improved integral coefficient model for assessing urban bus service regularity and to demonstrate that this model provides a more informative and realistic evaluation than the traditional single-indicator approach.

Literature Review

The problem of assessing bus service regularity has a long research history. One of the earlier foundational studies was conducted by Polus, who treated bus service reliability as a measurable operational characteristic rather than a purely descriptive service attribute. Later, Camus, Longo, and Macorini showed that automatic vehicle location data can be used to estimate transit reliability level-of-service more systematically. These studies were important because they established that reliability assessment should be based on quantitative operational evidence, not on general observations alone.

A major development in this field came with the transition from single punctuality indicators to richer reliability frameworks. Lin, Wang, and Barnum proposed a quality-control framework for bus schedule reliability, demonstrating that reliability should be monitored through a structured set of indicators derived from AVL data. Chen, Yu, Zhang, and Guo extended this logic by analyzing urban bus service reliability at the stop, route, and network levels, and by proposing multiple indicators such as route-based punctuality and stop-based deviation and evenness measures. These works clearly showed that service regularity is a multi-level phenomenon and cannot be fully represented by only one simple percentage indicator.

Another important direction in the literature concerns the broader assessment of transit service quality and performance through multidimensional approaches. Eboli and Mazzulla developed a methodology that combines subjective and objective service-quality measures, while Hassan, Hawas, and Ahmed proposed a multi-dimensional framework for evaluating transit performance at both system and route levels. Although these studies were not limited only to bus regularity, they are highly relevant because they support the methodological idea that transport performance should be evaluated through a combination of indicators rather than through one isolated metric. This idea is directly consistent with the logic of an integral coefficient model.

More specific advances in bus reliability measurement were made by studies that directly criticized threshold-based regularity indicators. Saberi, Zockaie, Feng, and El-Geneidy examined alternative bus service reliability measures at the stop level and showed that conventional measures do not always distinguish well between different operational situations, especially when early and late deviations have different passenger impacts. Gittens and Shalaby went further by explicitly evaluating bus reliability measures and proposing a new composite indicator. Their work is especially close to the present article because it confirms that composite indicators can reveal route performance differences that remain hidden when only a basic punctuality or regularity measure is used.

A parallel strand of the literature improved regularity assessment through richer operational data sources and more advanced statistical treatment. Uniman and co-authors used automated fare card data to measure service reliability from a passenger-oriented perspective. Barabino, Di Francesco, and Mozzoni later showed that the accurate treatment of AVL raw data substantially affects time-reliability measurement and also proposed an offline framework for diagnosing time-reliability problems across stops and time periods. Ma, Zhu, Koutsopoulos, and Ferreira added a travel-time reliability perspective by applying quantile regression to AVL and farecard data. Together, these studies demonstrate that route assessment becomes more informative when it incorporates headway deviation, travel-time variability, and data-based diagnosis rather than relying only on binary on-time classification.

Recent review and methodological studies confirm the same tendency. Tirachini and co-authors synthesized the literature on headway variability and showed that the field now recognizes multiple metrics, determinants, and service-quality effects related to unreliability. Zhou and co-authors also demonstrated that modern predictive methods can be used to evaluate urban bus service reliability across variable time horizons, indicating that reliability assessment is becoming increasingly data-intensive and dynamic. However, despite these advances, much of the literature still either focuses on one metric at a time, concentrates on stop-level or network-level diagnosis, or develops broad performance frameworks that are not easily translated into a simple operational score for route comparison.

Therefore, a clear research need remains for a compact and operationally interpretable route-level assessment model that integrates several essential dimensions of bus regularity into one coefficient. The present article addresses this need by combining the basic regularity coefficient, headway stability, travel-time stability, and trip completion into a single weighted integral index. In this sense, the contribution of the paper is not to replace the existing reliability literature, but to synthesize its most practically useful dimensions into a dissertation-oriented mathematical tool suitable for route monitoring, comparison, and managerial decision-making.

Problem Statement

Let N_p denote the number of planned trips during the observation period, and let N_r denote the number of trips performed within the admissible deviation range. The traditional regularity coefficient is determined as

$$K_r = \frac{N_r}{N_p}$$

or, in percentage form,

$$R_b = \frac{N_r}{N_p} \cdot 100$$

This coefficient is simple and operationally convenient. However, it reflects only one aspect of route performance, namely the share of regular trips. It does not reflect the following important factors:

- the magnitude of headway deviations;
- the stability of travel time along the route;
- the number of missed trips relative to the plan.

Because of this, the traditional coefficient may provide an incomplete picture of service quality. The problem, therefore, is to construct an improved model that includes several operational indicators simultaneously and combines them into one integral measure.

The improved coefficient must satisfy the following conditions. First, it must remain simple enough for practical use in route analysis. Second, it must include the most important operational components of regularity. Third, it must allow comparison of routes with different operating conditions. Fourth, its result must be interpretable both in coefficient form and in percentage form.

Research Method

The research method is based on the construction of a composite mathematical indicator. At the first stage, the regularity assessment problem was decomposed into several operational components. At the second stage, a normalized coefficient was defined for each component. At the third stage, these coefficients were combined into a weighted integral index.

Four components were selected for the improved model:

- the basic regularity coefficient;
- the headway stability coefficient;
- the travel time stability coefficient;
- the trip completion coefficient.

The average absolute headway deviation is determined by

$$\bar{\Delta}h = \frac{1}{m}\sum j = 1^m \mid h_j^{act} - h_p \mid$$

where h_j^{act} is the actual observed headway at control point j, h_p is the planned headway, and m is the number of observations.

The headway stability coefficient is then written as

$$K_h = 1 - \frac{\bar{\Delta}_h}{h_p}$$

Similarly, the average absolute travel time deviation is determined by

$$\bar{\Delta}t = \frac{1}{n}\sum i = 1^n \,|\, t_i^{act} - t_p \,|$$

where t_i^{act} is the actual travel time of trip i, t_p is the planned travel time, and n is the number of observed trips.

The travel time stability coefficient is

$$K_t = 1 - \frac{\bar{\Delta}_t}{t_p}$$

If N_m denotes the number of missed trips, then the trip completion coefficient is

$$K_m = 1 - \frac{N_m}{N_p}$$

After defining these four normalized coefficients, the improved integral regularity coefficient is constructed as

$$K_I = w_1 K_r + w_2 K_h + w_3 K_t + w_4 K_m$$

subject to the condition

$$w_1 + w_2 + w_3 + w_4 = 1$$

In percentage form, the integral regularity indicator is written as

$$R_I = K_I \cdot 100$$

For the pilot calculation in this paper, the following weights were adopted:

$$w_1 = 0{,}35,\ w_2 = 0{,}30,\ w_3 = 0{,}20,\ w_4 = 0{,}15$$

These weights reflect the assumption that the basic share of regular trips remains the most important component, while headway stability, travel time stability, and trip completion also contribute significantly to the total route assessment.

Model Development

The logic of the proposed model is based on the idea that service regularity is not a one-dimensional phenomenon. It is a combined result of schedule adherence, headway stability, temporal predictability, and completeness of route operation. Therefore, the integral coefficient gives a broader operational interpretation than the traditional indicator.

The traditional coefficient K_r answers the question of how many trips were regular. The coefficient K_h answers how stable the actual headway was relative to the planned interval. The coefficient K_t reflects how predictable the travel time was during the observation period. The coefficient K_m shows how fully the planned service was implemented without cancellations or omitted trips.

By combining these components, the proposed model distinguishes between routes that appear similar in basic percentage terms but differ substantially in operational stability. This is especially important for urban bus systems, where small disturbances accumulate along the route and where irregularity is often manifested not only through formally irregular trips, but also through unstable intervals and travel-time variability.

For illustration, consider a route with the following observed values:

$$N_p = 100,\ N_r = 89,\ h_p = 10{,}0,\ \bar{\Delta}_h = 2{,}4,\ t_p = 40{,}0,\ \bar{\Delta}_t = 7{,}6,\ N_m = 5$$

Then the component coefficients are

$$K_r = \frac{89}{100} = 0{,}89,\ K_h = 1 - \frac{2{,}4}{10{,}0} = 0{,}76,\ K_t = 1 - \frac{7{,}6}{40{,}0} = 0{,}81,\ K_m = 1 - \frac{5}{100} = 0{,}95$$

The integral coefficient is therefore

$$K_I = 0{,}35 \cdot 0{,}89 + 0{,}30 \cdot 0{,}76 + 0{,}20 \cdot 0{,}81 + 0{,}15 \cdot 0{,}95$$

$$K_I = 0,3115 + 0,228 + 0,162 + 0,1425 = 0,844$$

Hence, the improved regularity score in percentage form is

$$R_I = 0,844 \cdot 100 = 84,4\%$$

This example clearly shows that the traditional value of 89,0% is reduced to 84,4% when route instability is taken into account.

Computational Experiment

A computational experiment was carried out for five urban bus routes. For each route, the component coefficients were calculated and then aggregated into the improved integral coefficient.

The results are presented below.

Route	R_b **, %**	K_h	K_t	K_m	R_I **, %**
1	92,0	0,89	0,91	0,98	91,8
2	90,0	0,82	0,86	0,97	87,9
3	89,0	0,76	0,81	0,95	84,4
4	88,0	0,71	0,77	0,94	81,6
5	87,0	0,65	0,73	0,93	78,5

The experiment shows that the traditional regularity values decrease only slightly from route to route, namely from 92,0% to 87,0%. At first sight, this suggests that all routes perform relatively similarly. However, the improved integral coefficient shows a wider and more realistic separation of route quality, from 91,8% to 78,5%.

The difference is especially visible for Routes 3, 4, and 5. Although their traditional regularity values remain close to 89,0%, 88,0%, and 87,0%$, the integral scores are considerably lower because the routes experience substantial headway and travel-time instability.

For Route 5, the gap between the traditional and improved indicators is

$$87,0-78,5=8,5$$

percentage points.

For Route 4, the gap is

$$88,0-81,6=6,4$$

percentage points.

Thus, the improved model reveals operational weaknesses that remain hidden when only the basic percentage of regular trips is used.

Results and Discussion

The results of the computational experiment confirm that the traditional regularity coefficient is not sufficient for a full assessment of urban bus route performance. Its main weakness lies in excessive aggregation. It records whether the trip satisfies the admissible threshold, but it does not show how far the system has moved away from stable operation.

The integral coefficient solves this problem by combining several operational dimensions into one unified measure. This makes the assessment more sensitive to route instability. If the route operates with strong headway fluctuations, unstable travel times, or missed trips, the integral score decreases even if the traditional regularity percentage remains relatively high.

From a practical point of view, this is important for transport authorities and dispatch services. When only the basic coefficient is used, routes with hidden operational problems may be incorrectly classified as satisfactory. The improved coefficient provides a more balanced basis for managerial decisions, such as timetable revision, fleet redistribution, terminal control, and route restructuring.

Another advantage of the proposed model is its flexibility. The weighting coefficients may be adapted to local priorities. For example, if headway evenness is considered more important on high-frequency routes, the value of w_2 can be increased. If missed trips are particularly critical in a given network, a larger weight may be assigned to w_4. Thus, the structure of the model remains stable, while its calibration may be adjusted to the goals of the transport authority.

At the same time, the proposed model does not reject the traditional regularity coefficient. On the contrary, it uses that coefficient as one of the main components. Therefore, the model should be viewed as an improved and extended evaluation tool rather than as a completely different assessment philosophy.

Conclusion

This paper developed an improved integral coefficient model for assessing urban bus service regularity. Unlike the traditional single-indicator approach, the proposed model combines four operational components: the basic regularity coefficient, headway stability, travel time stability, and trip completion.

The computational experiment showed that the improved model provides a more informative and realistic assessment of route quality. In several cases, routes with relatively high traditional regularity values were shown to have substantially lower integral scores because of instability in headways, travel times, and trip completion.

The practical significance of the model lies in its applicability to route monitoring, comparative diagnostics, and planning analysis. It may be used by transport agencies to identify unstable routes, rank route performance more accurately, and support decisions on schedule improvement and operational control.

In future research, the model may be extended by including additional factors such as passenger load variability, stop dwell-time dispersion, and traffic congestion. However, even in its present form, the developed integral coefficient provides a strong analytical basis for improving the mathematical assessment of urban bus service regularity.

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